30. *On Finite Groups, Whose Sylow Groups Are All Cyclic.*

By Kinya HONDA.

(Comm. by Z. SUETUNA, M.J.A., May 12, 1949.)

In this paper we shall call those finite groups whose Sylow groups are all cyclic, *hypercyclic*. The group of order may also be called hypercyclic. The aim of this paper is to study the structure of these groups and then to find out all hypercyclic groups of a given order. The proofs of the results will be published afterwards elsewhere.

§ 1. Structure of Hypercyclic Groups.

When a group G has a subgroup A and a normal subgroup B, and $AB=G$ $AB=E$, where E is the unit subgroup of G, then we shall say that G *has a splitting into* A *and* B, and shall denote it by

$$G=(A,\ B\sigma_0).$$

Every element a of A gives rise to an automorphism τ $(xaxa,\ x \varepsilon B)$ of B. Let τ_a correspond to a, then we have a homomorphic mapping in the group of automorphisms of B from the group A. The σ in (1) should be understood as a symbol of this marpping. The elements of A, permutable with all elements of B, form a normal subgroup F of G, and will be called the *foundation group of the splitting* (1) The index $[A:F]$, if it is fnite will be called the *index of the splitting* (1).

Henceforth we shall study only finite groups. When afnite group G has a splitting into G_0 and P, where P is a cyclic group and at the same time a Sylow group of G, then we shall call it a *simple splitting* of G. Let F be the foundation group then G/F is cyclic. Now we have following theorems ;

Let

$$G=(G_0,\ P/\sigma)$$

be a simple splitting of G, *and* F *be the foundation group.*

1) **First Reduction Theorem on Subgroups.** *Any subgroup* H *can be expressible in a form*

$$H=a^{-1}\ HP'\ a,$$

mhere H *and* P *are subgroups of* G_0 *and* P *respectively, and* a *is an element of* P.

2) **Second Reduction Theorem on Subgroups.** *If*