27. *Fundamental Theory of Toothed Gearing (V).*

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In the preceding reports——(I) to (IV)——we have explained various properties of profile curves on a plane. Now we shall discuss profile curves on a sphere in this report (V) and in the following reports (VI) and (VII). As well as in the case of plane curves we confine ourselves to deal with such continuous spherical pitch or profile curves as at each of points on them a single tangent may be drawn continuously (although cusps are allowed to exist), and suppose that they make respectively one-point contact motion.

Almost all of the results which we have derived in the case of plane profile curves can be interpreted as the facts on a sphere by replacing a few words, for example——replacing the word "the tangent" on the plane to the word "the tangent great circle" on the sphere.

For the sake of simplicity in the following we shall say merely pitch or profile curves in place of spherical pitch or profile curves, if not needed.

§ 1. Necessary and sufficient conditions for profile curves (1).

As a necessary condition that two curves F_1 and F_2 invariably connected with two pitch curves K_1 and K_2 respectively be a pair of profile curves we have the following analogue of Descartes theorem for plane profile curves :

(α). *The common normal great circle to the curves F_1 and F_2 at any point of contact of them always passes through the common pitch point.*

From the condition (α) we obtain the following necessary and sufficient condition for profile curves.

Theorem 1. *A necessary and sufficient condition that two curves F_1 and F_2 invariably connected with two pitch curves K_1 and K_2 respectively be a pair of profile curves is that two perpendicular great cireles from any common pitch point to F_1 and F_2 coincide with each other in the direction and in the arc lngth to their feet.*

We shall say two families of small circles are developable from one upon another, if they consist of circles having centers at corresponding pitch points on K_1 and K_2 and epual spherical radii. Then we have

Theorem 2. *A necessary and sufficient condition that two curves*