54. Supplementary Remarks on Frobeniusean Algebras. I.

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(Comm. by Z. SUETUNA, M.J.A., July 12, 1949.)

The purpose of the present note is to make a supplementary remark to our previous structural criteria for Frobeniusean and quasi-Frobeniusean algebras.[1] The criteria were used to extend the notions to general rings with minimum condition, but our present remark is primarily concerned with the case of algebras[2] ratheh than rings with minimum condition. We show namely that in eacr of the criteria one half may be dropped in case of algebras. The result is applied to refine our previous result on residue-algebras. It may be used also to show that our previous criteria of Frobeniusean and quasi-Frobeniusean algebras in terms of duality of annihilation ideals may be reduced to one half too (in case of algebras, not rings with minimum condition);[3] this, together with some other results, we shall reserve for a succeeding joint note by M. Ikeda and myself.

Theorem 1. *Let A be an algebra over a field F, and N be its radical; the left annihilator $R = l(N)$ of N being the largest fully reducible right ideal of A. Suppose, now, that A possesses a left unit element E and that each right ideal $e_\kappa R$ is irreducible and isomorphic to $e_{\pi(\kappa)} A/e_{\pi(\kappa)} N$ where $\{e_\kappa(\kappa = 1, 2, \dots, k)\}$ is a maximal*

1) An algebra is called Frobeniusean if it possesses unit element and if its left and right regular representations are equivalent. It is called quasi-Frobeniusean if it possesses unit element and if the totalities of distinct components in its left and right regular representations coincide. For their structure cf. T. Nakayama, On Frobeniusean algebras, I, Ann. Math. 40 (1941); III, Jap. Journ. Math. 18 (1942). — The alluded criteria were given in I, § 2, Lemma 2; cf. also II, § 4.

The writer wants to take this opportunity to make up a miss in III, which Mr. O. Nagai has kindly pointed out to the writer. Namely, in the Corollary to Theorem 1 of the paper we have to make assumption of the existence of a unit element (or else a some assumption which assures that the radical be contained in every maximal left or right ideal.

2) Under an algebra we always mean an algebra with finite rank (over a ground field).

3) The present study has been given impulsion by this problem raised by M. Ikeda.