78. An Alternative Proof of a Generalized Principal Ideal Theorem.

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Recentry Mr. Terada[1] has proved the following generalized principal theorem :

Theorem. Let K be the absolute class field over k, and Ω a cycic intermediate field of K/k, then all the ambigous ideal classes of Ω will become principal in K.

I also generalized this theorem to the case of ray class field.[2]

By using Artin's law of reciprocity we can state above theorem in terms of the Galois group, and we have

Theorem. Let G be a metabelian group with commutator subgroup G', H be an invariant subgroup of G with the cyclic quotient group G/H, and A element of H with $ASA^{-1}S^{-1}\epsilon H'$ (S being a generator of G/H), then the "Verlagerung" $V(A) = \prod TA\overline{TA}^{-1}$ from H to G' is the unit element of G. Thereby T runs over a fixed representative system of G/H, and $\overline{TA}$ means the representative corresponding to the coset TAG'.

At first we tried to solve this by means of Iyanaga's method depending upon Artin's splitting group,[3] which is generated by G' and the symbols $A_\sigma (A_1 = 1,\ \sigma\epsilon\Gamma = G/G')$, and with Γ as operator system by rules

$$U^\sigma = S_\sigma U S_\sigma^{-1} \quad (U\epsilon G'), \tag{1}$$

$$A_\tau^\sigma = A_\sigma^{-1} A_{\sigma\tau} D_{\sigma,\tau}^{-1}, \tag{2}$$

S_σ being the representative of G/G' corresponding to $\sigma\varepsilon\Gamma$, and

$$D_{\sigma,\tau} = S_\sigma S_\tau S_{\sigma\tau}^{-1}. \tag{3}$$

But it seemed to us as if his method were not so easily applicable to our problem, and Terada at last checked the classical method of Furtwängler,[4] which brought him to success, after a rather complicated computation.

Here I will give a more simple proof, which depends upon Artin's splitting group.