62. Note on Pseudo-Analytic Functions.

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1. Let $w=f(z)=u(x, y)+iv(x, y)$, $z=x+iy$, be an inner transformation in the sense of Stoïlow in a connected domain D. Denote by E a set, in D, such that D and the derived set E' of E have no point in common. We suppose that

a) u_x, u_y, v_x, v_y exist and are continuous in $D^*=D-E$,

b) $J(z)=\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} > 0$ at every point in D^*,

c) the function $q(z)$ defined as the ratio of major and minor axes of an infinitesimal ellipse with centre $f(z)$, into which an infinitesimal circle with centre at every point z of D^* is transformed by $f(z)$, is bounded in $D^*: q(z) \leqq K$.

$f(z)$ is then called pseudo-meromorphic (K) in D[1].

The purpose of the present note is to give some results concerning pseudo-conformal representations and the cluster sets of pseudo-meromorphic functions.

2. Let $w=f(z)$ be pseudo-meromorphic (K) in a connected domain D. It is known that the set of $[z, w]$ where $w=f(z)$, $z\in D$, defines a Riemann surface Φ, in the sense of Stoïlow, spread over the w-plane. By the theory of uniformizations of P. Koebe, there exists a function $z'=\varphi(w)$ analytic in Φ which maps Φ on a plane (*schlicht*) domain D' of the z'-plane. Consequently we get a function $z(z')$ (or $z'(z)$) which defines a pseudo-conformal mapping (K) between D and D', by eliminating w from $w=f(z)$ and $z'=\varphi(w)$.

Thus we see that a function $w=f(z)$, pseudo-meromorphic (K) in D, is a composition of a uniform function $w=\varphi^{-1}(z')$, analytic in D' and a univalent function $z'(z)$, pseudo-regular (K) in D.

In view of the above consideration, it may be of some interest to investigate "*Verzerrungssatz*" concerning pseudo-conformal mapping (K). We first show that the properties of Fatou and Gross-Ahlfors hold for a bounded and univalent function, pseudo-regular (K).

Theorem 1. (Fatou's property). *Let* $w=f(z)=u(r, \theta)+iv(r, \theta)$, $z=re^{i\theta}$, *be*

1) S. Kakutani, *Applications of the theory of pseudo-regular functions to the type-problem of Riemann surfaces*, Jap. Journ. of Math., vol. 13 (1937), pp. 375–392.