56. A Generalization of the Principal Ideal Theorem.

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Furtwängler's principal ideal theorem[1] concerning the Hilbert's absolute class field K over k can be generalized, when the concerning class field is cyclic, in the following manner: There is only one ambiguous class in such case namely the principal class. This follows with usual manner by using group theoretical consideration as a simple application of Artin's law of reciprocity, or more directly from the formula for the ambiguous class number, or "Hauptgeschlechtssatz". We see namely, when K/k is a cyclic class field, every class C of K belongs to the principal genus, so that it is the $(1-s)$-th power of an ideal class, by the "Hauptgeschlechtssatz". It follows immediately that the correspondence $C \rightarrow C^{1-s}$ is an isomorphism, so that we can conclude $C=1$ from $C^{1-s}=1$.

One of the authors (Tannaka) has conjectured several years ago, that the above result is also true for the general abelian class field K/k, in the following sense:

Theorem. Let Ω be a cyclic intermediate field in the absolute class field K/k, then every ambiguous class of Ω/k becomes principal in K.

Group theoretical formulation of this theorem is so:

Theorem. Let G be a metabelian group with abelian commutator subgroup G′, H be an invariant subgroup of G with the cyclic quotient group G/H, and A an element of H with $ASA^{-1}S^{-1} \in H'$ (S being a generator of G/H), then the "Verlagerung" $V(A) = \Pi\ TA\ \overline{TA}^{-1}$ from H to G′ is the unit element of G. Thereby T runs over a fixed representative system of H/G′, and $\overline{TA}$ means the represenfative of the coset TAG′.

Tannaka has found this theorem to be true by several p-groups, but could not prove in general, and the problem remained long unsolved. Recently this problem was taken up again in our institute, and Terada succeeded at last to solve the problem. His method is in substance Furtwängler's classical proof, but needs further extremely complicated calculation. Iyanaga's elegant proof[2] refuses, at least in the present stage, its application to our problem. It seems to us, that our theorem allows further generalization, and is now under investigation.