# 55. On the Foci of Algebraic Curves.

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1. The points of intersection of tangents drawn from the imaginary circular points at infinity to an algebraic curve of the $n$-th class are called the Foci of the algebraic curves. As is known there are $n^2$ foci. Now we determine the locus of foci of algebraic curves in the pencil of algebraic curves of the $n$-th class, and make the extention of it in space.

2. Let us prove the dual theorem.

Theorem 1. *Supposing the points $P$, $Q$ to be intersections of variable algebraic curve in the pencil of algebraic curves of the $n$-th order with the two given straight lines $g$, $h$, the straight line $FQ$ envelops an algebraic curve of the $(2n-1)$-th class, which has the straight lines $g$, $h$ as $(n-1)$-ple tangents.*

Proof. In proving the above theorem, let us assume the equation of the pencil of algebraic curves of the $n$-th order to be

$$\sum_{i+j+k=n} A_{ijk} x^i y^j z^k = 0,$$

$$A_{ijk} = a_{ijk} + \lambda b_{ijk}$$

and the straight lines $g$, $h$

$$g; \qquad z = 0,$$

$$h; \qquad y = 0,$$

then the coordinates of the point $P$ $(x_1, y_1, 0)$ are given by

$$\sum_{i+j=n} A_{ij0} x^i y^j = 0,$$

and the coordinates of the point $Q$ $(x_1', 0, z_1')$ are given by

$$\sum_{i+k=n} A_{i0k} x^i z^k = 0.$$

Let us use line coordinates $u$, $v$, $w$ of the line $FQ$, then we have

$$u x_1 + v y_1 = 0,$$

$$u x_1' + w z_1' = 0.$$

Hence

$$\sum_{i+j=n} A_{ij0} (-1)^i v^i u^j = 0,$$

$$\sum_{i+k=n} A_{i0k} (-1)^i w^i u^k = 0.$$

Eilminating $\lambda$, from both equations, we get