1. Equilibrium Potentials and Energy Integrals.

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1. Introduction

Frostman's theory[1] on equilibrium potentials of order α has been recently extended by Kunugui[2] to generalized potentials. The purpose of this paper is to study the same problem from another point of view.

For preparation we state some definitions on generalized potentials and capacities. Denote by Ω the whole Euclidean space, by $\delta(E)$ the diameter of a bounded Borel set E, by r_{PQ} the length of a segment PQ, and by D_E^m the family of non-negative mass distributions of total mass m on a bounded Borel set E; especially when $m=1$, we denote it by D_E simply. Let $\Phi(t)$ be a strictly monotone decreasing and continuous function defined in the interval $(0,\infty)$ such that $\lim\limits_{t\to 0}\Phi(t)=+\infty$. Given any mass distribution μ on E, we call the Lebesgue-Stieltjes integrals

$$\int_E \Phi(r_{PQ})d\mu(Q) \quad\text{and}\quad \iint_E \Phi(r_{PQ})d\mu(Q)d\mu(P)$$

the Φ-potentials and the Φ-energy integrals respectively with respect to μ. Put

$$V_E^\Phi=\inf_{\mu\in D_E}\sup_{P\in\Omega}\int_E\Phi(r_{PQ})d\mu(Q) \quad\text{and}\quad W_E^\Phi=\inf_{\mu\in D_E}\iint_E\Phi(r_{PQ})d\mu(Q)d\mu(P),$$

then it is easily seen that

$$\Phi[\delta(E)]\leqq V_E^\Phi\leqq+\infty \quad\text{and}\quad \Phi[\delta(E)]\leqq W_E^\Phi\leqq+\infty.$$

We define the Φ-capacity $C^\Phi(E)$ of E as follows; if $V_E^\Phi<+\infty$, then $C^\Phi(E)=\Phi^{-1}[V_E^\Phi]$, and if $V_E^\Phi=+\infty$, then $C^\Phi(E)=0$, where Φ^{-1} denotes the inverse function of Φ. Hereafter we shall write for the sake of simplicity V_E, W_E, $C(E)$ for V_E^Φ, W_E^Φ, $C_E^\Phi(E)$.

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1) O. Frostman: Potentiel d'équilibre et capacité des ensembles avec quelques applications à la théorie des fonctions. Thèse. Lund. 1935.

2) K. Kunugui: Sur quelques points de la théorie du potentiel. (I), (II). Proc. Jap. Acad. vol. 21-23.