24. On the Theory of Semi-Local Rings.

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Introduction.

The concept of local ring was introduced by Krull [7][1]. That of semi-local ring, a generalization of local ring, was introduced by Chevalley [1]. It was defined namely as a Noetherian ring R possessing only a finite number of maximal ideals. If $\mathfrak{m}$ denotes the intersection of all maximal ideals in a semi-local ring R, then $\bigcap_{n=1}^{\infty}\mathfrak{m}^n=(0)$, and so, R becomes a topological ring with $\{\mathfrak{m}^n\}$ as a system of neighbourhoods of zero. Chevalley derived many properties by making use of the concept of ring of quotients introduced by Grell [5]. He also introduced, in [2], a generalization of ring of quotients, in order to generalize Proposition 8, § II, [1]. But this generalization was only with respect to a Noetherian ring and the complementary set of a prime ideal. A further, and very natural, generalization of the concept of ring of quotients was given by Uzkov [6]. But it seems to me that also this generalization is not convenient to be applied to a generalized theory of semi-local rings which I want to present in the following. So we first introduce, after a short discussion of Uzkov's ring of quotients, a notion of topological quotient ring, which constitutes Chapter I. In Chapter II, we introduce semi-local rings in our generalized sense. They enjoy, besides some other properties, most of the propositions in [1]; an exception is the assertion that R is a complete semi-local ring with the intersection $\mathfrak{m}$ of all maximal ideals and if R' is a ring such as (1) R' contains R as a subring and (2) $\bigcap_{n=1}^{\infty}\mathfrak{m}R'=(0)$, then there exists $m(n)$ for each n such as $\mathfrak{m}^{m(n)}R'\cap R\subseteq\mathfrak{m}^n$ (a part of Proposition 4, II, 1). Appendix gives some supplementary remarks concerning our generalized notions.

We list the correspondences between the assertions in the present paper and those in [1, § II] or [3, Part I]:

Throughout this paper, a ring means a commutative ring with the identity element. Under a subring we mean a subring having the same identity. We will say that α is integral over a ring R if α satisfies a suitable monic equation with coefficients in R. θ denotes the empty set.

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1) The number in brackets refers to the bibliography at the end.