26. *On the Behaviour of the Boundary of Riemann Surfaces, II.*

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In Part I of the same title[1] we have dealt with the absolute harmonic measure of the boundary of Riemann surfaces and defined the kind of their connected pieces. In this paper we will investigate chiefly the behaviour of the boundary of Riemann surfaces of the first kind.

§ 1. Some theorems on the boundary of Riemann surfaces of the first kind.

Theorem 1. Let $\bar{F}$ be a connected piece of the first kind with the relative boundary (Γ^i) and the proper boundary E of a Riemann surface F spread over the z-plane and $\mathfrak{F}_\rho$ be any connected piece of $\bar{F}$, whose boundary includes no point of (Γ^i), lying above a disc K: $|z-a|<\rho$, where a is an arbitrary point on the z-planes. Then $\mathfrak{F}_\rho$ covers any point inside of K at least once, except a set of points of capacity zero.[2]

Proof. (i) The boundary of $\mathfrak{F}_\rho$ is both E and Jordan curves (γ), whose projections on the z-plane coincide with the circumference γ: $|z-a|=\rho$.

At this time we will prove that there exists on $\mathfrak{F}_\rho$ no non-constant bounded harmonic function $v(z)$, which has the next condition:

$$v(z)=0 \text{ on } (\gamma), \qquad 0\leqq v(z)\leqq 1 \text{ on } \mathfrak{F}_\rho .$$

Suppose that $v(z_0)>0$, $z_0\in\mathfrak{F}_\rho$.
We consider approximating function $\bar{u}_n(z)$ and let $z_0\in\bar{F}_n$.
At this time let $\mathfrak{F}_\rho^{(n)}$ one of the cross-cuts $\mathfrak{F}_\rho\cdot\bar{F}_n$, which includes z_0.
Then the boundary of $\mathfrak{F}_\rho^{(n)}$ is both (γ) and (C_n^j).
Since from above

$$v(z)=0 \text{ on } (\gamma), \qquad v(z)\leqq 1 \text{ on } (C_n^j),$$
$$\bar{u}_n(z)\geqq 0 \text{ on } (\gamma), \qquad \bar{u}_n(z)=1 \text{ on } (C_n^j),$$

then by the maximum principle

$$v(z)\leqq\bar{u}_n(z) \text{ on } \mathfrak{F}_\rho^{(n)} \text{ and } v(z_0)\leqq\bar{u}_n(z_0).$$

Since for $n\to\infty$ $\bar{u}_n(z)\to\bar{u}(z)\equiv 0$, $v(z_0)$ must be zero, which is absurd, q.e.d.

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1) Y. Nagai: On the Behaviour of the Boundary of Riemann Surfaces, I. Proc. Jap. Acad., vol. 26 (1950).

2) In this paper, "capacity" means the logarithmic capacity.