39. On Conformal Slit Mapping of Multiply-Connected Domains.

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1. We choose, as a basic domain of standard type in the theory of conformal mapping of $n\ (\geqq 3)$-ply connected domains, a concentric circular ring cut along $n-2$ disjoint concentric circular slits, and denote the boundary components of such a domain, D_Q say, by

$$C^{(1)}: |z| = 1; \qquad C^{(2)}: |z| = Q\ (<1);$$
$$C^{(j)}: |z| = m_j, \quad \theta_j \leqq \arg z \leqq \theta_j' \ (3 \leqq j \leqq n).$$

Each domain bounded by $n\ (\geqq 3)$ disjoint continua possesses $3n-6$ (real) conformal invariants as its *moduli*. For instance, the moduli of the circular slit annulus D_Q may be given by the $3n-6$ quantities

$$Q,\ m_j\ (3 \leqq j \leqq n),\ \theta_j - \theta_3\ (3 < j \leqq n),\ \theta_j' - \theta_3\ (3 \leqq j \leqq n).$$

Let D_{Q_0} be a domain on the w-plane conformally equivalent to D_Q and obtained from a circular slit annulus of the same type as D_Q by cutting along a slit (Jordan arc) Γ_{Q_0} which starts from a point on the exterior boundary component $|w| = 1$. An extremal property given by Rengel[1] shows that the radius Q_0 of the interior boundary component of D_{Q_0} never exceeds Q and is, moreover, always less than Q provided D_{Q_0} does not coincide with D_Q. Let now the function mapping D_Q schlicht and conformally onto D_{Q_0} in such a way that the both peripheral boundary circumferences correspond each other, be denoted by

$$w = f(z), \qquad f(Q) = Q_0;$$

this mapping function is uniquely determined under the additional condition explicitly written here.

In case of simply-connected domains, the *Löwner's differential equation* for slit mapping has been recognized as a very fruitful instrument in the theory, a brief proof of which may be given by making use of Poisson formula for functions regular analytic in a circle.[2] This equation can also be generalized to the doubly-con-

1) E. Rengel: Über einige Schlitztheoreme der konformen Abbildung. Schriften d. Math. Sem. u. Inst. f. angew. Math. d. Univ. Berlin **1** (1932/3), 141-162. Cf. also H. Grötzsch: Über einige Extremalprobleme der konformen Abbildung, I. Leipziger Berichte **80** (1928), 367-376.

2) Y. Komatu: Über einen Satz von Herrn Löwner. Proc. Imp. Acad. Tokyo **16** (1940), 512-514.