38. *Note on the Envelope of Regularity of a Tube-Domain.*

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§1. Introduction.

In the space of n complex variables $(z_1,\ldots, z_n)$, there exists a domain B, such that *any* function analytic in B has an analytic continuation over the domain D which is strictly larger than B. Such D is called an *analytic completion* of B. For any domain B, there corresponds a domain $\mathbf{H}(B)$ called its *envelope of regularity*, or *maximal analytic completion*, such that[1]

(i) $\mathbf{H}(B)$ is an analytic completion of B, and

(ii) $\mathbf{H}(B)$ is a domain of regularity, i.e. there exists a function which cannot be continued beyond $\mathbf{H}(B)$.

The geometrically explicit form of the envelope of regularity for a given domain still remains almost unknown. One of the few results concerning this branch is the following due to S. Bochner[2]:

Theorem 1. *The envelope of regularity of a tube-domain T is its convex hull (convex closure) $\mathbf{C}(T)$. Here the tube-domain means the point set which can be written in the form*

$$(1)\quad T = \{(z_j=x_j+iy_j) \mid (x_1,\ldots, x_n \in S, \mid y_j \mid < \infty, \ (j = 1,\ldots, n)\}.$$

where S is a domain in the real n-dimensional space $(x_1,\ldots,x_n)$, and S is called the base of T.

It seems quite natural that this theorem should be conjectured from the facts that the mapping $w_j = \exp z_j$ transformes T into a covering surface over a Reinhardt domain in (w_j)-space, and that the Reinhardt domain of regularity is convex in logarithmic sense. But his original proof is based upon the expansion of the

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1) P. Thullen: Die Regularitätshüllen. Math. Ann. **106** (1932) 64–76.
H. Cartan–P. Thullen: Regularitäts- und Konvergenzbereiche. Math. Ann. **106** (1932) 617–647.

2) S. Bochner: A theorem on analytic continuation of functions in several variables. Annals of Math. **39** (1938) 14–19.
S. Bochner–W.T. Martin. Several complex variables. Princeton 1948, Chap. V.

3) Cf. e.g. H. Cartan: Les fonctions de deux variables complexes et le problème de la représentation analytique.