56. *On the Zeros of Dirichlet's L-Fuhctions.*

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We put $h = \varphi(k)$ where k is a positive integer and $\varphi(k)$ is Euler's function. Let $\chi(n)$ denote one of the h Dirichlet's characters with modulus k. $\bar{\chi}$ is the conjugate complex character of χ. $\zeta(s,w)$ and $L(s,\chi)$ denote the functions defined for $\sigma > 1$ by $\sum_{n=0}^{\infty}(n+w)^{-s}$ and $\sum_{n=1}^{\infty}\chi(n)n^{-s}$ respectively, where $0 < w \leqq 1$ and $s = \sigma + ti$. Throughout the paper, the notations $A \ll B$ and $A = O(B)$ for $B > 0$ show that $|A| \leqq KB$, where K is a positive absolute constant.

We know from the recent work of *Rodosskiĭ* ([11], Theorem 1.) that the number of $L(s,\chi)$ which have a zero in the rectangle

$$1-\frac{\psi(k)}{\log kT} \leqq \sigma \leqq 1, \quad |t-T_1| \leqq K\log^2 kT$$

where $\frac{1}{4}\log k \geqq \psi(k) \geqq \log\log k$ and $T = |T_1|+2$ does not exceed $B\exp(A\,\psi(k)+5\log\log kT)$. From this we are able to deduce that the total number of zeros of all the L-functions with modulus k in the above rectangle does not exceed

$$C\exp(A\,\psi(k)+8\log\log kT) \tag{1}$$

where A, B, C and K are positive absolute constants.

The aim of this paper is to estimate the total number $N(\alpha, T)$ of zeros of all the L-functions with modulus k in the rectangle

$$\alpha \leqq \sigma \leqq 1, \quad |t| \leqq T$$

using Ingham's method [7]. The main result is that, if

$$\zeta(\tfrac{1}{2}+ti, w) - w^{-\frac{1}{2}-ti} = O(|t|^c) \tag{2}$$

where c is a positive absolute constant, then

$$N(\alpha, T) = O\{(k^4T^{4c}(T+k)^2)^{1-\alpha}\log^8 kT\}$$

for $\frac{1}{2} \leqq \alpha \leqq 1$, $T \geqq 2$. From this we are also able to deduce (1) and so *Rodosskiĭ's* main theorem ([11], Theorem 2.) in the theory of primes in an arithmetic progression.

We use some well known theorems in the theory of functions in the following forms.

Theorem A. (*Jensen*, [6], Theorem D, p. 49.) Suppose that