69. *On the Behaviour of a Pseudo-Regular Function in a Neighbourhood of a Closed Set of Capacity Zero.*

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Since R. Nevanlinna[1] established many important theorems on the behaviour of a meromorphic function in a neighbourhood of a closed set of capacity zero, many results on this problem have been obtained. M. Tsuji[2] has found that Evans' theorem plays an important role in such investigations and obtained many theorems systematically.

Pseudo-regular functions were first studied by H. Grötzsch[3]. He has proved that Picard's theorem can be generalized to a class of pseudo-regular functions and his theorem has been extended by M. Lavrentieff[4].

We have never known the study on the behaviour of a pseudo-regular function in a neighbourhood of a closed set of capacity zero, so that we treat with this problem. The object of this paper is to obtain generalizations of the theorems of Nevanlinna such as an extension of Liouville's theorem and that of the principle of maximum to a class of pseudo-regular functions.

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1.[5] A uniform continuous function

$$w = f(z) = u(x, y) + iv(x, y),\ z = x + iy$$

is called pseudo-regular in a domain, if it satisfies the following three conditions

(1) u_x, u_y, v_x, v_y exist and are continuous in the domain.

(2) $J(z) = u_x v_y - u_y v_x > 0$ except possibly at most the countable set of points which has no point of accumulation inside of the domain.

(3) At the point z_0 where $J(z_0) = 0$, a sufficiently small neighbourhood of z_0 is transformed topologically on a neighbourhood of an algebraic branch point at $w_0 = f(z_0)$ of a Riemann surface.

A transformation by a pseudo-regular function is called to be

1) R. Nevanlinna, Eindeutige analytische Funktionen (1936), pp. 130–136.
2) M. Tsuji, Jap. Jour. of Math. 19 (1944).
3) H. Grötzsch, Leipziger Berichte, 80 (1929).
4) M. Lavrentieff, C. R. 200 (1935).
5) S. Kakutani, Jap. Jour. of Math. 13 (1937), O. Teichmüller, Deutsche Math. 3 (1938.)