17. An Alternative Proof of Liber's Theorem.

By Shigeo SASAKI.

Mathematical Institute, Faculty of Science, Tôhoku University.

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§ 1. Introduction.

In Mathematical Review 11 (1950), Prof. M. S. Knebelmann communicated the following results of Liber (Doklady Akad. Nauk SSSR (N.S), 66 (1949)) concerning the structure of affinely connected and Riemannian spaces with one-parametric holonomy groups.

Theorem A. *Suppose that the holonomy group H of a given affinely connected space A_n be a one parametric group. If we denote the symbol of the infinitesimal transformation of H by $Xf = a^i_j x^j \frac{\partial f}{\partial x^i}$ (a^i_j : const.), then the rank of the matrix $\| a^i_j \|$ is at most 2.*

Theorem B. *Suppose that the holonomy group H of a given Riemannian space V_n be a one parametric group. Then V_n admits $(n-2)$ parallel vector fields which are independent each other. Accordingly, V_n is the product space of a two dimensional Riemannian space and an $(n-2)$ dimensional Euclidean space.*

I shall give here alternative proofs of Liber's theorems. Although I can not see his paper, it is certain that my proof is quite different from his original proof. Perhaps my proof will be more geometrical than his proof.

§ 2. Riemannian spaces.

We shall first state *Cartan's Lemma.* Suppose that the holonomy group of a nonholonomic space E with the fundamental group G be g. Then we can choose frames at each point of E so that the connexion of the space in consideration is analytically the same as those of a space with the fundamental group g.

When we are going to apply this Lemma to Riemannian and affinely connected spaces, we must note that the word "holonomy group" is used in different senses in introduction and in Cartan's Lemma. The holonomy group in introduction is the so called "homogeneous holonomy group" that is the group of linear homogeneous transformations belonging to the holonomy group in ordinary sense.