16. On the Simple Extension of a Space with Respect to a Uniformity. I.

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In the present and the next notes we shall develop a general theory concerning the simple extension of a space with respect to a uniformity. As special cases we obtain various topological extensions of spaces such as completions of uniform spaces in the sense of A. Weil[1] (or more generally in the sense of L. W. Cohen[2]) and the bicompact extensions of T-spaces due to N. A. Shanin[3] (a generalization of Wallman's bicompactification).

§ 1. **Definitions.** In the present note we say that R is a *space*, if R is an aggregate of "points" and to each subset A of R there corresponds a set $\bar{A}$, called the closure of A, with the following properties:

1) $A \subset \bar{A}$, 2) $\bar{\bar{A}} = \bar{A}$,
3) $A \subset B$ implies $\bar{A} \subset \bar{B}$, 4) $\bar{0} = 0$.

Thus R is a neighbourhood space such that we can take as a basis of neighbourhoods of a point p a family of open sets containing p. As is well known a space which satisfies the additivity of the closure operation: $\overline{A+B} = \bar{A}+\bar{B}$ is a T-space.

Let R be a space. A collection $\{\mathfrak{U}_\alpha ; \alpha \in \Omega\}$ of open coverings of R is called a *uniformity*. Two uniformities $\{\mathfrak{U}_\alpha\}$ and $\{\mathfrak{V}_\lambda\}$ are called *equivalent*, if for any $\mathfrak{U}_\alpha \in \{\mathfrak{U}_\alpha\}$ there exists a covering $\mathfrak{V}_\lambda \in \{\mathfrak{V}_\lambda\}$ which is a refinement of $\mathfrak{U}_\alpha$, and conversely for any $\mathfrak{V}_\lambda$ there exists $\mathfrak{U}_\beta \in \{\mathfrak{U}_\alpha\}$ such that $\mathfrak{U}_\beta$ is a refinement of $\mathfrak{V}_\lambda$. We say that a uniformity $\{\mathfrak{U}_\alpha ; \alpha \in \Omega\}$ *agrees with the topology*, if it satisfies the condition:

(A) $\{S(p, \mathfrak{U}_\alpha) ; \alpha \in \Omega\}$ is a basis of neighbourhoods at each point p of R.

1) A. Weil: Sur les espaces a structure uniforme et sur la topologie générale, Actualites Sci. Ind. **551**, 1937; J. W. Tukey: Convergence and uniformity in topology, 1940.

2) L. W. Cohen: On imbedding a space in a complete space, Duke Math. J. **5** (1939), 174–183.

3) N. A. Shanin: On special extensions of topological spaces, Doklady URSS, **38** (1943), 3–6; On separation in topological spaces, ibid., 110–113; On the theory of bicompact extensions of topological spaces, ibid., 154–156. These papers are not yet accessible to us. We knew the results by Mathematical Reviews.