# 15. On the Behaviour of Power Series on the Boundary of the Sphere of Analyticity in Abstract Spaces.

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In classical analysis there exists a singular point at least on the circle of convergence of the power series, but this is not true generally in the case of the power series in complex Banach spaces. In this paper we shall investigate a necessary and sufficient condition for power series in complex Banach spaces to be analytic at all points on the boundary of the sphere of analyticity.

Let $E$ and $E'$ be two complex Banach spaces and an $E'$-valued function $h_n(x)$ defined on $E$ be a homogeneous polynomial of degree $n$. Then the radius of analyticity of the power series $\sum_{n=0}^{\infty}h_n(x)$ exists, which is written by $\tau$*). The sphere $\|x\|<\tau$ is called the sphere of analyticity of the power series $\sum_{n=0}^{\infty}h_n(x)$.

**Theorem 1.** In order that $\sum_{n=0}^{\infty}h_n(x)$ is analytic at all points on the boundary of the sphere of analyticity, it is necessary and sufficient that

$$\overline{\lim_{n\to\infty}}\sqrt[n]{\sup_{x\in G}\|h_n(x)\|}<\frac{1}{\tau}, \quad \cdots\cdots\cdots\cdots \quad (1)$$

for an arbitrary compact set $G$ extracted from the set $\|x\|=1$.

**Proof.** Let $\sum_{n=0}^{\infty}h_n(x)$ be analytic at all points on $\|x\|=\tau$. If a compact set $G$ extracted from $\|x\|=1$ exists and it satisfies the following equality

$$\overline{\lim_{n\to\infty}}\sqrt[n]{\sup_{x\in G}\|h_n(x)\|}=\frac{1}{\tau},$$

we have

$$\frac{1}{\tau+\varepsilon_i}<\sqrt[n_i]{\sup_{x\in G}\|h_{n_i}(x)\|} \quad \cdots\cdots\cdots\cdots \quad (2)$$

for a sequence of positive numbers $\varepsilon_1>\varepsilon_2>\ldots>\varepsilon_n>\ldots$ which tends to zero and for $n_i$ which corresponds to $\varepsilon_i$, where $i=1, 2,\ldots$

Since $G$ is compact, there exists $x_i$ in $G$ which satisfies $\sup_{x\in G}\|h_{n_i}(x)\|=\|h_{n_i}(x_i)\|$. Then we can select from $\{x_i\}$ a subsequence which converges to $x_0$ and, of course, $x_0\in G$. In order not