14. *On the Type of an Open Riemann Surface.*

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(Comm. by K. KUNUGI, M.J.A., Feb. 12, 1951.)

1. Let F be an open abstract Riemann surface and Γ be its ideal boundary. Suppose that $F_n (n = 0, 1, \ldots)$ is the relatively compact (shlicht or non) subdomain of F satisfying the following four conditions:

1°) F_o is simply and $F_n (n \neq 0)$ is finitely connected,

2°) $F_n \subset F_{n+1}$,

3°) if Γ_n is the boundary of F_n, Γ_n consists of a finite number of analytic closed curves and $\Gamma_n \frown \Gamma_{n+1} = 0$,

4°) $\bigcup_{n=0}^{\infty} F_n = F$.

Putting $R_n = F_n - \bar{F}_o$, the boundary of R_n consists of Γ_n and Γ_o. Let P be the inner point of R_n and denote by $\omega_n = \omega_n (\Gamma_n, P, R_n)$ the harmonic measure of Γ_n at P with respect to the domain R_n. Then we call

$$D(\omega_n) = \iint_{R_n} \left[\left(\frac{d\omega_n}{dx}\right)^2 + \left(\frac{d\omega_n}{dy}\right)^2\right] dxdy, \qquad t = x + iy,$$

the Dirichlet integral of ω_n with respect to the domain R_n, where t is the local parameter.

R. Nevanlinna [3] has proved the following:

Theorem. The ideal boundary Γ of the Riemann surface F is of harmonic measure zero if and only if $\lim_{n\to\infty} D(\omega_n) = 0$.

2. Let u be the harmonic function in the domain R_n such that

$$u = \begin{cases} 0 & \text{on } \Gamma_o, \\ \log \mu_n & \text{on } \Gamma_n \qquad (\mu_n > 1), \end{cases}$$

and, if v is the conjugate harmonic function of u, then the total variation on Γ_o equals to 2π, i.e.,

$$\int_{\Gamma_o} dv = 2\pi .$$

In this case we call $\log \mu_n$ the modul of the domain R_n.

We shall show the following:

Theorem 1. Let $\log \mu_n$ *be the modul of* R_n *and* ω_n *be the harmonic measure of* Γ_n *with respect to* R_n. *Then we have*