13. *On a Locally Compact Group with a Neighbourhood Invariant under the Inner-automorphisms.*

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(Comm. by K. KUNUGI, M.J.A., Feb. 12, 1951.)

Prof. H. Freudenthal[1)] proved that a locally compact connected group with an arbitrarily small neighbourhood invariant under the inner-automorphisms is isomorphic to the direct product of a vector group and a compact group.

As an extention of the above theorem the author will prove the following:

Theorem. *Let G be a locally compact connected group with a fixed neighbourhood U. If U is invariant under the inner-automorphisms, then G contains a compact normal subgroup N such that G/N is isomorphic to the direct product of a vector group and a compact group.*

Proof. Let m be the left-invariant Haar measure with a real-valued function $\Delta(x)$ on G such that for an open set V,

$$m(Vx) = m(V)\Delta(x).$$

Then $\Delta(x) = 1$ because

$$m(U)\Delta(x) = m(Ux) = m(xU) = m(U).$$

We see therefore that m is at the same time right-invariant.

Without loss of generality we may assume that U is regularly open.

Put

$$N = \{x;\ m(xU \cup U - xU \cap U) = 0\}.$$

Then N coincides with the set $\{x;\ xU = U\}$ since U is a regularly open set. Clearly N is a closed subgroup. Furthermore N is compact and normal since $N \subset UU^{-1}$ and

$$a^{-1}xaU = a^{-1}xUa = a^{-1}Ua = U.$$

Let us introduce a metric $d(X, Y)$[2)] into the factor group G/N by

$$d(X, Y) = m(xU \cup yU - xU \cap yU).$$

Clearly this metric is left-invariant. Moreover this is right-invariant, for