38. *The Two-sided Representations of an Operator Algebra.*

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The object of the present note is to investigate the relation between the two-sided representations and the traces of a uniformly closed operator algebra on a Hilbert space (i.e. C*-algebra in the tirminology of I.E. Segal [7]). Our investigation is closedly connected with the recent works of R. Godement [2], I. E. Segal [9] and J. Dixmier [1].

1. We suppose that R is a C*-algebra *having the identity* 1 (with elements x, y, z, etc.) and ω, σ, τ etc. are the *states* of R (i. e. line functionals on R, considering as a Banach space, with $\omega(xx^*) \geqq 0$ for all x and $\omega(1) = 1$). A *trace* of R is a state which satisfies moreover $\tau(xy) = \tau(yx)$ for any pair x and y. If for any x there exists a trace τ such that $\tau(xx^*) > 0$, then we say that R has *sufficiently many traces* (or shortly is of the *trace type*). The *state space* of all states is a convex and weakly* closed subset in the unit sphere of the conjugate space of R. Also it is easy to see that the set T (the *trace space*) of all traces forms a convex and weakly* closed subset in the state space. Whence by the well-known theorem of Tychonoff, they are compact in the (bounded) weak* topology of the conjugate space. It is an easy consequence of the theorem due to M. Krein and D. Milman [3] that *a* C*-*algebra has sufficiently many traces if and only if it has sufficiently many characters* where we mean by a *character* an extreme point of the trace space.

Concerning the notion of the trace type, the following observation may have some interest. If the "Poisson bracket" $[x, y] = i(xy-yx)$ of any pair of hermitean clements x and y is not strictly positive definite then we will call that the algebra is of *semi-trace type*. This terminology is justified by the following

THEOREM 1. *A C*-algebra is of semi-trace type if and only if it has at least one trace.*

Since the proof of this theorem can be done in somewhat similar manner to that of our preceeding paper [5], we may omit the detail.

2. Let now H be a Hilbert space with elements ξ, η, ζ, etc. In this space we now introduce the following