36. *Remark on a Set of Postulates for Distributive Lattices.*

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1. Introduction.

G. Birkhoff gives the following set of postulates for distributive lattices:(*)

Any algebraic system which satisfies

(1) $a \frown a = a$ *for all* a ,

(2) $a \smile I = I \smile a = I$ *for some* I *and all* a ,

(3) $a \frown I = I \frown a = a$ *for some* I *and all* a ,

(4) $a \frown (b \smile c) = (a \frown b) \smile (a \frown c)$

and $(b \smile c) \frown a = (b \frown a) \smile (c \frown a)$, *for all* a, b, c ,

is a distributive lattice with I .

G. Birkhoff proposes as the Problem 65 l.c. the following question: *Prove or disprove the independence of the seven identities assumed as postulates in Theorem 3.*

We shall remark first, that the system of axioms, as given above, is not sufficient to define the distributive lattices. If, indeed, we denote with I_2 one of the elements I in (2) and with I_3 one of I in (3), it may happen that $I_2 \neq I_3$, as the following example shows:

$\smile$	I_2	I_3
I_2	I_2	I_2
I_3	I_2	I_3

$\frown$	I_2	I_3
I_2	I_2	I_2
I_3	I_2	I_3

this system satisfies all the axioms (1)-(4), and is not a distributive lattice.

However, we may take sets of postulates, quite analogous to the one given above, to define a distributive lattices. We propose in the following lines four kinds of such postulate-sets, (I)-(IV). Any algebraic system, satisfying any one of these sets, turns out to be a distributive lattice with I. Each set consists of four, five or six postulates, which we shall prove as independent. Thus the Problem 65 of Birkhoff may be considered as solved.