105. Probability-theoretic Investigations on Inheritance. III$_4$. Further Discussions on Cross-Breeding.

(Further and Ultimate Continuation.)

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5. Continuous process.

In preceding sections we have considered a stepwise process of cross-breeding and determined an explicit form of general formulae representing distribution in nth generation. In the present section we shall idealize the process such that it proceeds continuously with the lapse of the time. According to such a manner of treatment, the sums or differences of various kinds in the previous discussion will then be replaced by the corresponding integrals or derivatives. It will also be recognized that several formulae will be considerably clarified by such an idealization.

We have considered, in §3, the general class $X'^{2^n-u}X''^{u}$ $(0 \leqq u \leqq 2^n)$ appearing in the nth generation, which possesses the frequency of the gene given by (3.10), namely by

$$2^{-n}((2^n-u)p_i'+up_i'') = (1-2^{-n}u)p_i'+2^{-n}up_i'' \quad (i=1,\dots,m).$$

When the discrete variable n restricted to integral values runs from 0 to 2^n, this frequency varies from p_i' to p_i''. Correspondingly, we introduce a frequency of the gene A_i given by

$$(5.1) \qquad p_i(x) \equiv (1-x)p_i'+xp_i'' \qquad (i=1,\dots,m),$$

depending linearly on a continuous variable x running over the closed interval $0 \leqq x \leqq 1$, where $\{p_i'\}$ and $\{p_i''\}$ denote two given fixed distributions; x being a quantity which corresponds to $2^{-n}u$ in the previous discrete case. Evidently, it holds always

$$\sum_{i=1}^{m} p_i(x) = (1-x)\sum_{i=1}^{m} p_i' + x\sum_{i=1}^{m} p_i'' = (1-x)+x = 1.$$

The frequency $p_i(x)$ is also contained in the interval between p_i' and p_i'', both ends inclusive.

We further introduce the variable t representing the time, and denote by $\delta(x;t)$ the population-density at the time t. Supposing, for the sake of brevity, that the total population remains constant, it will be expressed by

$$(5.2) \qquad \Delta = \int_0^1 \delta(x;t)\,dx.$$