104. *Probability-theoretic Investigations on Inheritance. III₃. Further Discussions on Cross-Breeding.*

(Further Continuation.)

By Yûsaku KOMATU.

Department of Mathematics, Tokyo Institute of Technology and
Department of Legal Medicine, Tokyo University.

(Comm. by T. FURUHATA, M.J.A., Oct. 12, 1951.)

4. Consequences of main result.

In the last section we have proved the main result, stating

$$\bar{A}_{ii}(n)-\bar{A}_{ii}(0) = -\Gamma^{(n)}(p_i'-p_i'')^2, \qquad (i, j = 1,\ldots, m\,;\, i<j)\ ,$$
$$\bar{A}_{ij}(n)-\bar{A}_{ij}(0) = -2\Gamma^{(n)}(p_i'-p_i'')(p_j'-p_j'')$$

where

$$\Gamma^{(n)} = \sum_{l=1}^{n-1}\frac{1}{2^{2l-1}}\sum_{k=1}^{2^{l}-1}k^2\sum_{h=0}^{2^{l}-1-k}\mu_{h,h+k}^{(l)}+\frac{1}{2^{2n-2}}\sum_{k=1}^{2^{n}-1}k^2\sum_{h=0}^{2^{n}-1-k}\mu_{h,h+k}^{(n)}.$$

Having been established this result, we can attempt the similar considerations on the comparison between $\bar{A}_{ii}(n)$ or $\bar{A}_{ij}(n)$ and the corresponding quantities with respect to earlier generations, as done in a special case in §2.

First, for instance, if $p_i' = p_i''$ for some fixed suffix i, we see that the relations

$$(4.1) \qquad \bar{A}_{ij}(n) = \bar{A}_{ij}(0) \qquad (j = 1,\ldots, m\,;\, n = 1, 2,\ldots)$$

are always valid, regardless of the frequencies of the remaining genes; the relations $\bar{A}_{ij}(n) = \bar{A}_{ij}^{*}\ (j = 1,\ldots, m)$ hold then also good. We thus conclude that, if the sub-races X' and X'' being composed into X possess an identical frequency of a gene A_i for some i, then every genotpye partaken by A_i remains constant with respect to generation, regardless of the frequencies of the remaining genes, in how manner the cross-breeding does take place. Here, it must, of course, be remembered that the condition of equilibrium states of both sub-races is kept to be assumed.

Naturally, in probability-theoretic treatment of general theory on inheritance phenomena, an immediate object of various considerations is the frequencies of the genes, and hence those of genotypes. Consequently, the immediately deducible concrete results concerning phenotypes refer only to phenotypes consisting of the genes dominant against no other genes, as O or M and N in ABO or MN blood types, respectively.

However, the above obtained result makes possible to deduce a general result also on phenotypes. In fact, as noticed above, if