102. *Probability-theoretic Investigations on Inheritance.* *III_1. Further Discussions on Cross-Breeding.*

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(Comm. by T. FURUHATA, M.J.A., Oct. 12, 1951.)

1. Preliminaries.

In two previous papers[1] we have developed a general theory of inheritance of a character consisting of m distinct genes denoted by A_i $(i = 1, \ldots, m)$, the inheritance of which is assumed to be subjected to Mendelian law. Especially, in §4 of I and §1 of II, we have promised to discuss the case in detail where the buffer effect grows gradually through several generations. In the present paper we shall treat such a problem. For the sake of brevity, we shall now confine ourselves to consider a population X composed of *two* sub-races X' and X''. Suppose that these two sub-races are initially both *in equilibrium states*, then denoting the frequencies of genes A_i by p'_i and p''_i $(i = 1, \ldots, m)$, the frequencies of phenotypes are then given by the fromulae

$$(1.1)\qquad \begin{cases} \bar{A}'_{ii} = p'^2_i, \\ \bar{A}'_{ij} = 2p'_i p'_j; \end{cases} \quad \begin{cases} \bar{A}''_{ii} = p''^2_i, \\ \bar{A}''_{ii} = 2p''_i p''_j \end{cases} \quad (i, j = 1, \ldots, m;\ i < j).$$

Suppose further that two races X' and X'' are mixed at a rate $\lambda' : \lambda''$ $(\lambda' + \lambda'' = 1)$, then the frequencies of the A_i in the limit distribution of X are, in view of the gereral result (1.7) of II, given by

$$(1.2)\qquad p_i = \lambda' p'_i + \lambda'' p''_i \qquad (i = 1, \ldots, m),$$

and hence those of genotypes in the limit distribution of X are then, because of (1.8) of II expressed in the form

$$(1.3)\qquad \bar{A}^*_{ii} = (\lambda' p'_i + \lambda'' p''_i)^2,\ \bar{A}^*_{ij} = 2(\lambda' p'_i + \lambda'' p''_i)(\lambda' p'_j + \lambda'' p''_j)\ (i \neq j).$$

On the other hand, let the initial distribution of X, i.e., the distribution of X at the moment of mixture, be denoted by $\bar{A}_{ij}(0)$ $(i \leqq j)$, while in the previous paper II it was denoted merely by $\bar{A}_{ij}$ (cf. (1.9) of II). By general relations established in (1.9) of II_1, we then obtain

$$(1.4)\qquad \begin{aligned} \bar{A}_{ii}(0) &= \lambda' \bar{A}'_{ii} + \lambda'' \bar{A}''_{ii} = \lambda' p'^2_i + \lambda'' p''^2_i, \\ \bar{A}_{ij}(0) &= \lambda' \bar{A}'_{ij} + \lambda'' \bar{A}''_{ij} = 2(\lambda' p'_i p'_j + \lambda'' p''_i p''_j) \end{aligned} \qquad (i, j = 1, \ldots, m;\ i < j).$$

1) Y. Komatu, Probability-theoretic investigations on inheritance. I. Distribution of genes; II. Cross-breeding phenomena. Proc. Jap. Acad., **27** (1951), I, 371–377; II, 378–383, 384–387. These will be referred to as I and II respectively.