89. *Determination of a 3-Cohomology Class in an Algebraic Number Field and Belonging Algebra-Classes.*

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(Comm. by Z. SUETUNA, M.J.A., Oct. 12, 1951.)

Let k be an algebraic number field (of finite degree) and K/k be a (finite) Galois extension with Galois group $\mathfrak{K}$. Let I_K, P_K be the groups of idèles and principal idèles in K. The class field theory gives rise to a factor set, of $\mathfrak{K}$, in the factor group of the idèle-class group $\mathfrak{C}_K = I_K/P_K$ modulo its component of unity. This factor set can be represented by a certain factor set in the idèle-class group $\mathfrak{C}_K$ itself which satisfies some further requirements, as was shown by Weil [7]; a different, direct derivation of the same factor set has been given in Nakayama [4], [5]. This last factor set in $\mathfrak{C}_K$ is called a canonical factor set for K/k, and is determined uniquely by K/k in the sense of equivalence. Let $\{\mathfrak{a}(\sigma, \tau)\}$ $(\sigma, \tau \in \mathfrak{K})$ be a such canonical factor set for K/k and $a(\sigma, \tau)$ be idèles which represent the idèle-classes $\mathfrak{a}(\sigma, \tau)$. Then the coboundary $\alpha = \delta a$ (given by $\alpha(\rho, \sigma, \tau) = a(\sigma, \tau)a(\rho\sigma, \tau)^{-1}a(\rho, \sigma\tau)a(\rho, \sigma)^{-\tau}$) is a 3-cochain in P_K and is in fact a 3-cocycle. In this way we have a 3-cohomology class α in P_k attached in invariant manner to K/k. The order of this 3-cohomology class α has been determined in [5] and is equal to the degree $(K:k)$ divided by the least common multiple of p-degrees of K/k, p running over all primes in k.

On the other hand, if $\mathfrak{A}$ is a central simple algebra over K such that every $\sigma \in \mathfrak{K}$ can be extended to an automorphism of $\mathfrak{A}$, then $\mathfrak{A}$ determines a certain 3-cohomology class in P_K, called the Teichmüller class of $\mathfrak{A}$ ([6]). MacLane [3] has shown that the totality of the 3-cohomology classes arising in this way (with different $\mathfrak{A}$'s) form a cyclic group of the same order as that of α described above. In fact, it was shown by Hochschild and the writer that α is a generator of this cyclic group ([2]).

Now arises the problem to determine the exact algebra-class (though not unique) whose Teichmüller-class is (not only a power (with exponent prime to the above order) of, but) exactly our α, attached invariantly to K/k. The answer is given by the following theorem: Let n_p be the p-degree of K/k, for a prime p in k, and let n' be the least common multiple of all the n_p, p running over all primes in k. Then $\mathfrak{A}$ *has* α *as its Teichmüller-class if, and only*