130. *Probability-theoretic Investigations on Inheritance.* IV$_3$. *Mother-Child Combinations.*

(Further Continuation.)

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(Comm. by T. FURUHATA, M.J.A., Nov. 12, 1951.)

3. Mother-children combination concerning families with two childeren.

We have hitherto been interested in combinations consisting of a mother with her one child. Now, we can treat similar problem on those of a mother with her two children. We shall call two children, the order of which is also taken into account, briefly the first child and the second child. But, they may be, in general, any two childeren chosen among their brethren or sisters; it will be noticed that they are not necessarily the first-born and the second-born in the strict sense. In this two-children case, various new interesting results will be derived in comparison with the previous one-child case. Moreover, the results on the latter case are contained in the present case as special ones. *Two children belonging to the same family mean, in the following, those having both parents in common.* Now, both children belonging to the same family, or more generally those having mother alone in common will possess the types which are not quite independent each other but between which certain correlation is existent. In fact, genotype of each child must then contain at least one gene in common with that of mother. If, in particular, mother is homozygotic, genotypes of both children contain always at least one gene, namely the one composing the type of mother, in common each other.

Now, we consider an inheritance character which consists of m genes $A_i(i=1,\dots,m)$ with distribution-probability $\{p_i\}$, the distribution being supposed to be in an equibrium state. The number of permutations, admitting the repetition, of selecting any two types of children without kinship is evidently equal to $\frac{1}{4}m^2(m+1)^2$. On the other hand, that of selecting any two children having a common mother is equal to m^2 or $(2m-1)^2$ according to the mother of homozygote or of heterozygote, respectively. If they are further restricted such as to have a father also in common, then, the number of possible permutations reduces to a small number. In fact, as seen from the table in §3 of I, we get the following table.

Mating	$A_{ii}\times A_{ii}$	$A_{ii}\times A_{ik}$	$A_{ii}\times A_{hh}$	$A_{ii}\times A_{hk}$	$A_{ij}\times A_{ij}$	$A_{ij}\times A_{ik}$	$A_{ij}\times A_{hk}$
Permutation	1	4	1	4	9	16	16