129. *Probability-theoretic Investigations on Inheritance.* *IV$_2$. Mother-Child Combinations.*

(Continuation.)

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2. Mixed mother-child combination.

We have considered in the previous section a unique population in which an inherited character is distributed in an equilibrium state. In the present section we shall consider two populations X and X' each of which has an equilibrium distribution with respect to the inherited character. Every mating between X and X' will produce a half-breed. Thus, a problem arises, to determine the corresponding probabilities of mother-child combinations for such a cross-breeding case.

Let now the distributions of genes in X and X' be denoted by $\{p_i\}$ and $\{p_i'\}$, respectively. To fix an idea, we suppose that in any mating its mother and father are chosen from X and X', respectively. Then, the table, corresponding to that for pure breeding given in §3 of I, becomes as follows in the next page, the convention that the suffices i, j, h, k are different each other is here also made.

In the present case, the order of members in each mating must be taken into account. Accordingly, except $\frac{1}{2}m(m+1)$ kinds of matings between the coinciding types, each of remaining matings in the previous table must be divided into two. Thus, the total number of combinations is, as inserted in the table, equal to

$$(2.1) \qquad 2\,\tfrac{1}{8}m(m+1)(m^2+m+2)-\tfrac{1}{2}m(m+1)=\tfrac{1}{4}m^2(m+1)^2\,;$$

this number is nothing but the square of that of possible genotypes of the inherited character under consideration.

We now denote by $\pi'(A_{ij}\,;A_{hk})$ or briefly by

$$(2.2) \qquad \pi'(ij\,;hk) \qquad (i,j,h,k=1,\ldots,m)$$

the probability of appearing of a combination of a mother A_{ij} with her child A_{hk}. Again $\pi'(ij\,;hk)$ is equal to zero provided (1.2) holds. The symmetry relations corresponding to (1.3) remain here also valid; namely,

$$(2.3) \qquad \pi'(ij\,;hk)=\pi'(ji\,;hk)=\pi'(ij\,;kh)=\pi'(ji\,;kh)$$