119. *On Unramified Extensions of Algebraic Function Fields.*

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Let K be an algebraic function field over an algebraically closed constant field k, with an arbitrary characteristic p. We can take a suitable element x of K such that K is separably algebraic over $k(x)$. The set $\mathfrak{L}_K$ of all differentials of the first kind of K forms a linear set over k whose dimension g is equal to the genus of K. Let L be a normal extension of K of degree n, $\mathfrak{G}$ the Galois group of this normal extension; let, further, $\mathfrak{L}_L$ be the linear set of all differentials of the first kind of L, whose dimension G is equal to the genus of L, d the degree of the different of L/K, then we have the well known formula of Hurwitz:

$$2G-2 = n(2g-2)+d. \tag{1}$$

L is clearly separably algebraic over $k(x)$, and every differential ω of L is expressed uniquely as $ydx\,(y\in L)$. ω is a differential of K if and only if $y\in K$. Let σ be an arbitrary element of $\mathfrak{G}$, then σ transforms ω to $\omega^\sigma = y^\sigma dx$. ω^σ depends only on ω and σ and not on the choice of a "separating element" x of K. Therefore σ induces a linear transformation of $\mathfrak{L}_L$, and if we choose a basis $\omega_1, \dots, \omega_G$ of $\mathfrak{L}_L$, we have a matric representation $\sigma \rightarrow \mathbf{A}(\sigma)$ of $\mathfrak{G}$ such as

$$\begin{pmatrix} \omega_1 \\ \vdots \\ \omega_G \end{pmatrix} = \mathbf{A}(\sigma) \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_G \end{pmatrix}. \tag{2}$$

We shall study the structure of this representation in the following lines. When the characteristic p of k is 0, this problem was solved completely by C. Chevalley and A. Weil.[1)]

If p is a prime number, Weil's proof may be extended to the case $(p, n) = 1$, but if p divides n, our problem is more difficult, and has not yet been solved in general. This difficulty arises from the fact that the representation (2) is not completely reducible. In this paper, we shall deal with a special case: L is unramified over K and $\mathfrak{G}$ is a cyclic group. In this case, the different of L/K is the unit divisor, and the genus G of L is equal to $n(g-1)+1$. Let σ be a generator of $\mathfrak{G}$, fixed once for all. We shall first solve our problem in two spacial cases.

1) C. Chevalley—A. Weil, Über das Verhalten der Integrale 1. Gattung bei Automorphismen des Funktionenkörpers. Abh. Math. Sem. Hamburg 10 (1934).