118. *Complete Continuities of Linear Operators.*

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1. A linear operator T mapping a Banach space E into itself is, according to F. Riesz (K. Yosida [14), (*weakly*) *completely continuous* provided that T carries the unit sphere into a (weakly) compact set. These operators, as it is well-known, play an important role in the theory of abstract integral equations (mean ergodic theorems).

Concerning weakly completely continuous operators, V. Gantmacher [5] proved, corresponding to Schauder's Theorem, that both T and its conjugate operator T^* are weakly completely continuous. He shows further, assuming a separability condition, the weak complete continuity is derivable by a condition concerning the values of its second conjugate T^{**}.

The first aim of the present note is to show that Gantmacher's conditions are equivalent without assuming the separability. This is carried out in Theorem 1 with the use of the Moore-Smith convergence of elements of Banach spaces which is introduced by L. Alaoglu [1]: A phalanx or a directed set of $x_\alpha \varepsilon E$ ($f_\alpha \varepsilon E^*$) *converges weakly* (*weakly**) to $x \varepsilon E$ ($f \varepsilon E^*$) if and only if $\{x_\alpha\}$ ($\{f_\alpha\}$) is bounded and $f(x_\alpha)$ ($f_\alpha(x)$) converges in the sense of Moore-Smith to $f(x)$ for all $f \varepsilon E^*$ ($x \varepsilon E$), where E^* will mean the conjugate space of E. This convergence will determine a topology of E (E^*) (cf. Alaoglu [1], Bourbaki [3], Tukey [13]). It will be called this topology as the *weak* (*weak**) *topology* of E (E^*). It is known that *the unit sphere of the conjugate space is compact with respect to the weak* topology* (cf. Alaoglu [1], Bourbaki [2], Kakutani [7]).

In the connection with Gantmacher's Theorem, a similar formulation for strong complete continuity will be expected. It is possible to do combining the theorems due to J. Schauder [12] and I. Gelfand [6], and will be formulated in Theorem 2. Although the proof is already known, it will be given a short proof, basing on a compactness theorem due to I. Gelfand [6]. (The proof here employed, including that of the compactness theorems of Gelfand and Phillips, is taken from a letter of Shûichi Takahashi, who send it to the author in the middle of 1949). It is to be noted that his proof of Schauder's theorem is closedly connected in some sense to a recently published proof of S. Kakutani [8]. The author expresses his thanks to S. Takahashi for the permission of the publication in the present note).