113. *Remarks on the Topological Group of Measure Preserving Transformation.*

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I. **Introduction.** Let I be the unit interval and m be the Lebesgue measure. Let G be the group of all measure preserving transformations of I onto itself. For any $S \varepsilon G$, measurable set A, and positive number ε, define neighbourhood $N(S)$ of S as follows:

$$N(S) = N(S, A, \varepsilon) = \{T : m(S(A) \ominus T(A)) < \varepsilon, \quad m(S^{-1}(A) \ominus T^{-1}(A)) < \varepsilon\}.$$

With this topology G is a complete topological group. The purpose of this note is to prove the following two properties of G.

Theorem 1. *G is simple,. i.e. G contains no closed normal subgroup except G and the identity E of G.*

Theorem 2. *G is arcwise connected.*

II. **Preliminaries.** The following definitions and results of P. R. Halmos[1] are used in the sequel.

1. A measure preserving transformation T is called nowhere periodic if $m\{x : x \varepsilon I, \ T^n x = x \ \text{ for some } n\} = 0.$[2]

2. If both T and S have exactly the same period n, then T and S are conjugate.[3]

3. The conjugate class of any nowhere periodic measure preserving transformation is everywhere dense in G.[4]

III. **Proof of Theorem 1.** Let us denote by N any closed normals ubgroup of G.

Lemma 1. If N contains a transformation of period n[5], $n \geqslant 2$, then N contains a nowhere periodic transformation.

Proof. We shall prove this lemma in three steps: (i) $n=2$, (ii) $n=3$ and (iii) $n \geqslant 4$.

(i) $n=2$. Let S and T be the transformations $Sx = -x$ and $Tx = -x + \gamma$ where γ is an irrational number, then both S and T are of period 2 and $Rx = STx = x + \gamma$[6]. By 2 of II both S and T

1) P. R. Halmos: In general a measure preserving transformation is mixing, Ann. of Math., 45, 1944, pp. 786–792.

2) P. R. Halmos, loc. cit. p. 787.

3) P. R. Halmos, loc. cit. p. 789.

4) P. R. Halmos, loc. cit. p. 789.

5) In this note we shall call T to be of period n when T has exactly the period n.

6) Cf. P. R. Halmos and J. von Neumann: Operator methods in classical mechanics II, Ann. of Math., 43, 1942, pp. 332–350.