# 3. On Rings of Operators of Infinite Classes.

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Let $\boldsymbol{M}$ be a ring of operators in a Hilbert space $\boldsymbol{H}$ in the sense of J. von Neumann [3], and denote the center of $\boldsymbol{M}$ by $\boldsymbol{M}^\natural$. Recently J. Dixmier has proved the following theorem [1; Theorems 10 and 11]:

**Theorem of Dixmier.** *If $\boldsymbol{M}$ is of finite class, then there exists a mapping $A \to A^\natural$ of $\boldsymbol{M}$ on $\boldsymbol{M}^\natural$ possessing the following properties:*

(1) *If* $A \in \boldsymbol{M}^\natural$, $A^\natural = A$,

(2) $(\lambda A)^\natural = \lambda A^\natural$,

(3) $(A+B)^\natural = A^\natural + B^\natural$,

(4α) $(AB)^\natural = (BA)^\natural$,

(4β) $(AB)^\natural = AB^\natural$ *if* $A \in \boldsymbol{M}^\natural$,

(5α) *If* $A \in \boldsymbol{M}_s$ *and* $A \geqq 0$, *then* $A^\natural \in \boldsymbol{M}_s$ *and* $A^\natural \geqq 0$,

(5β) *If* $A \in \boldsymbol{M}_s$, $A \geqq 0$ *and* $A^\natural = 0$, *then* $A = 0$,

(6) $(A^*)^\natural = (A^\natural)^*$.

*Furthermore, if there exists a mapping $A \to \varphi(A)$ of $\boldsymbol{M}$ on $\boldsymbol{M}^\natural$ with the properties* (1) (2) (3) (4α) *and* (5α), *then $\varphi(A) = A^\natural$ for all $A \in \boldsymbol{M}$.*

The present paper is a continuation of the one of Dixmier [1], and our object is to generalise the notion of his $\natural$-operation for the rings of operators of infinite classes. If $\boldsymbol{M}$ is a factor, our results include the one of Neumann [4].

We shall use the usual definitions and notations in the theory of rings of operators without any explanation, and the results of Dixmier will be assumed. The reader is reffered to [1] or [3].

1. Following [1] and [3], we shall say that a projection $E \in \boldsymbol{M}$ is *finite* if, for any projection $F \in \boldsymbol{M}$ $E \sim F$, $F \leqq E$ implies $F = E$, and *infinite* if this is not the case. If the unit element $I$ is finite, then we say $\boldsymbol{M}$ is *of finite class*, and otherwise $\boldsymbol{M}$ is *of infinite class.*

Consider those operators $A \in \boldsymbol{M}$ which are permutable with a projection $E \in \boldsymbol{M}$ and form their parts in $E$, $A_{(E)}$. Denote the set of all those $A_{(E)}$ ($A \in \boldsymbol{M}$, and permutable with $E$) by $\boldsymbol{M}_{(E)}$. We say $A \in \boldsymbol{M}$ is *contained* in $E$ if $AE = EA = A$. Then obviously $\boldsymbol{M}_{(E)}$ is a ring of operators in $E\boldsymbol{H}$ and $(\boldsymbol{M}_{(E)})^\natural = (\boldsymbol{M}^\natural)_{(E)}$ [3; Lemmas 11. 3. 2 and 11. 3. 4].