2. On the Algebraic Construction of the Picard Variety.

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(Comm. by Z. SUETUNA, M.J.A., Jan. 12, 1952.)

Introduction. In this paper, we shall show a purely algebraic construction of two Abelian Varieties called the "Picard Variety" and the "Albanese Variety" attached to a given complete normal Variety V^r for the universal domain of *arbitrary characteristic*. In the classical case, having the complex number field as the universal domain, in spite of the deep theories of Castelnuovo, Picard and Lefschetz, the distinction of the above two Abelian Varieties had been remained somewhat uncertain. Recently J. Igusa[1) has distinguished clearly these two Abelian Varieties and investigated the relations between them on the rigorous foundations of the theory of modern algebraic geometry and on the theory of harmonic differentials. On the duality theorem for Abelian Varieties and the theory of correspondences, we hope to study on some future occasions.

Let V^r be a normal projective Variety. W. L. Chow–v. d. Waerden's result on the associated-forms enable us to define algebraic families of positive cycles on V.[2), 3)] According to this, there is a bunch $\mathfrak{F}$ in a projective space such that, every positive V-divisor of the given degree is in a one-to-one correspondence with the Point of a component of $\mathfrak{F}$. Let $\{X\}$ be the totality of V-divisors corresponding to a component U of $\mathfrak{F}$. We call $\{X\}$ as a (*maximal*) *algebraic family* and U the *associated-Variety* of $\{X\}$. The Point of U corresponding to a member X of it is called the *Chow-Point* of X. Let $G_a(V)$ be the group generated by all the V-divisors $X-X'$, where X and X' belong to the same algebraic family. Any V-divisor belonging to $G_a(V)$ is called *algebraically equivalent to O.*

Let L be a generic linear Variety over a field of definition K for V such that $V. L$ is a Curve C. We shall say that C is a generic 1-section of V over K. C has no singular Point.[4)]

We shall use freely the results and terminologies in Weil's "Foundations of Algebraic Geometry", Amer. Math. Soc. Colloq., vol. 29, (1946) and "Variétés Abéliennes et Courbes Algébriques", Act. Sci. et Ind. (1948).

1) Forthcoming in the American Journal.

2) Cf. W.L. Chow-v.d. Waerden: "Zur algebraischen Geometrie IX". Math. Ann. 113.

3) Cf. W. L. Chow: "Algebraic system of positive cycles in an algebraic variety". Amer. Journ. vol. LXXII.

4) Cf. Y. Nakai: "On the section of an algebraic Variety by the generic hyperplane". Mem. Col. of Sci., Univ. Kyoto, Ser. A. XXVI. 1951.