1. A Note on Symmetric Algebras.

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The main purpose of the present note is to prove the following theorem[1] by a new method.

Theorem 1. *An algebra A over an algebraically closed field is symmetric if and only if its basic algebra is symmetric.*

As an application, we can show that absolutely uni-serial algebras are symmetric.

In what follows we assume always that A is an algebra with unit element over an algebraically closed field K. Let $S(a)$ and $R(a)$ be the left and the right regular representations of A, formed by means of a basis (u_i). A is called a Frobenius algebra if $S(a)$ and $R(a)$ are similar:

$$S(a) = P^{-1}R(a)P. \tag{1}$$

In particular, A is called a symmetric algebra when the matrix P can be chosen as a symmetric matrix[2].

Let $A = A^* + N$ be a splitting of an algebra A into a direct sum of a semisimple subalgebra A^* and the radical N of A. We shall denote by

$$A^* = A_1^* + A_2^* + \cdots\cdots + A_n^*$$

the unique splitting of A^* into a direct sum of simple invariant subalgebras. Let $e_{\kappa, \alpha\beta}$ $(\alpha, \beta = 1, 2, \ldots, f(\kappa))$ be a set of matrix units for the simple algebra A_κ^*. We set $e = \sum e_{\kappa, 11}$. Then eAe is an algebra with unit element e, which is called the *basic algebra*[3] of A. As one can easily see, the radical of eAe is $eAe \frown N = eNe$ and eAe/eNe is direct sum of fields.

Let now

$$A = A_1 \supset A_2 \supset \cdots\cdots \supset A_t \supset (0) \tag{2}$$

be a composition series for A considered as an (A, A) space. Then corresponding to (2), we obtain a composition series for eAe considered as an (eAe, eAe) space

$$eAe = eA_1e \supset eA_2e \supset \cdots\cdots \supset eA_te \supset (0) \tag{3}$$

1) See Nesbitt and Scott [5] p. 549.

2) Nesbitt and Nakayama [4].

3) Nesbitt and Scott [5].