30. *Probability-theoretic Investigations on Inheritance. VII$_6$. Non-Paternity Problems.*

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7^{bis}. Distribution of maximum probability.

We consider the case of mixed combination given by (4.12), i.e.,

$$P'=1-2S'_2+S'_3-2S_{1,1}^2+2S_{2,2}+3S_{1,1}S_{1,2}-3S_{2,3}. \tag{7.17}$$

The problem is to maximize this quantity under accessory conditions

$$0\leqq p_i,\ p'_i\quad (i=1,\ldots,m);\qquad \textstyle\sum_{i=1}^m p_i=\sum_{i=1}^m p'_i=1. \tag{7.18}$$

The set of maximizing distributions $\{p_i\}$ and $\{p'_i\}$, if existent interior to the ranges, would be determined by a system of equations

$$\begin{aligned}&\frac{\partial}{\partial p_i}\Big(P'-\lambda\Big(\sum_{j=1}^m p_j-1\Big)-\lambda'\Big(\sum_{j=1}^m p'_j-1\Big)\Big)=0,\\ &\frac{\partial}{\partial p'_i}\Big(P'-\lambda\Big(\sum_{j=1}^m p_j-1\Big)-\lambda'\Big(\sum_{j=1}^m p'_j-1\Big)\Big)=0\end{aligned}\qquad (i=1,\ldots,m),$$

$$\sum_{i=1}^m p_i=\sum_{i=1}^m p'_i=1;$$

λ and λ' denoting the Lagrangean multipliers. The first $2m$ equations become

$$\begin{gathered}p'_i(-4S_{1,1}+4p_ip'_i+3S_{1,2}+3p'_iS_{1,1}-6p_ip'^2_i)=\lambda,\\ -2p'_i+3p'^2_i-4p_iS_{1,1}+4p^2_ip'_i+3p_iS_{1,2}+6p_ip'_iS_{1,1}-9p^2_ip'^2_i=\lambda'\\ (i=1,\ldots,m).\end{gathered}$$

However, as suggested by the previously discussed special case $m=2$, it seems that the maximum of P' will rather be attained by an extreme distribution of $\{p_i\}$ lying on the boundary of its range; namely,

$$p_i=1\quad (i=i_0),\qquad p_i=0\quad (i\neq i_0) \tag{7.19}$$

for any value of $i_0\,(1\leqq i_0\leqq m)$. For such a distribution, P' becomes

$$P'_*=1-2S'_2+S'_3, \tag{7.20}$$

the value being independent of i_0.

The maximum of P'_* under the condition $\sum p'_i=1$ is surely attained by the symmetric distribution

$$p'_i=1/m\qquad (i=1,\ldots,m). \tag{7.21}$$