29. *Probability-theoretic Investigations on Inheritance. VII*$_5$. *Non-Paternity Problems.*

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6. Discontinuity on probablity based upon recessive genes.

As remarked in § 1, the probability of proving non-paternity is, in general, less in case upon phenotypes alone than in case upon the corresponding genotypes, provided that there exist recessive genes. Besides this trivial fact, we notice here a further remarkable fact which has probably not yet been explicitly noticed. We shall discuss in the present section this apparently curious phenomenon, which may be called the *discontinuity on probability of proving non-paternity based upon recessive genes.*

Now, if, in the formula (5.3), representing the whole probability of proving non-paternity with the aid of *ABO* blood type, we put $r=0$ and correspondingly $p+q=1$, then it becomes

$$\varphi \equiv [P_{ABO}]^{r=0} = qp^4 + pq^4 = pq(p^3+q^3) = pq(1-3pq). \tag{6.1}$$

On the other hand, if, in the corresponding formula (5.1), representing that with the aid of *MN* blood type, we merely substitute p and q instead of s and t respectively, then it becomes

$$\psi \equiv [P_{MN}]^{(s,t)=(p,q)} = pq(1-pq). \tag{6.2}$$

Hence, φ is less than ψ in general, i.e., surely so unless pq vanishes; in fact, the difference is equal to

$$\varphi - \psi = -2p^2q^2 \leqq 0. \tag{6.3}$$

In general, the comparison between the modes of inheritance of *ABO* and *MN* blood types immediately shows that, if particularly the gene *O* would be lacking, then the mode of inheritance of the former consisting of both genes *A* and *B* alone would reduce essentially to that of the latter. Consequently, it would superficially be expected as plausible that, if, in any general result on *ABO* blood type, we put particularly $r=0$ and replace s, t instead of p, q respectively, then a corresponding result on *MN* blood type would be obtained. Although this is really the case for the most part, all does not go well. A counter-example is offered by non-paternity problem under consideration. If r vanishes, the probability P_{ABO}