27. *Probability-theoretic Investigations on Inheritance. VII$_3$. Non-Paternity Problems.*

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3. Sub-probabilities of proving non-paternity.

We have derived, in the preceding section, a formula (2.20) representing the whole probability of proving non-paternity by composing the sub-probabilities for various types of mothers. Though somewhat superfluous, we may think that the whole probability is composed of sub-probabilities concerning various kinds of mother-child combinations. We shall discuss, in the present section, such a decomposition.

We first consider the partial sum of probabilities of proving non-paternity, generally denoted by (2.2), corresponding to mother-child combination both consisting of homozygotes, necessarily of the same types. In view of the first relation (2.11), we get

$$\sum_{i=1}^{m} P(ii;\, ii) = \sum_{i=1}^{m} p_i^3 (1-p_i)^2 = S_3 - 2S_4 + S_5. \tag{3.1}$$

Next, the partial sum corresponding to those consisting of homozygotic mothers and of heterozygotic children is, in view of (2.13), given by

$$\begin{aligned}\sum_{i=1}^{m} \sum_{j\neq i} P(ii;\, ij) &= \sum_{i=1}^{m} p_i^2 (1-2S_2+S_3-p_i(1-p_i)^2)\\ &= S_2(1-2S_2+S_3)-(S_3-2S_4+S_5) = S_2-S_3-2S_2^2+2S_4+S_2S_3-S_5.\end{aligned} \tag{3.2}$$

The partial sum corresponding to mother-child combinations consisting of heterozygotic mothers and of homozygotic children is given by the sum of the first two terms of the left-hand side of (2.16). Each summand being symmetric with respect to suffices i and j, we can apply the general formula (1.7) and then obtain

$$\begin{aligned}\sum_{i,j}{}' (P(ij;\, ii)+P(ij;\, jj)) &= \sum_{i,j=1}^{m} P(i\mathring{j};\, ii) - \sum_{i=1}^{m} P(i\mathring{i};\, ii)\\ &= \sum_{i,j=1}^{m} p_i^2 p_j (1-p_i)^2 - \sum_{i=1}^{m} p_i^3 (1-p_i)^2\\ &= S_2-2S_3+S_4-(S_3-2S_4+S_5) = S_2-3S_3+3S_4-S_5.\end{aligned} \tag{3.3}$$

Here, the notation analogous to (2.19) has been used; namely,