26. *Probability-theoretic Investigations on Inheritance.* *VII$_2$. Non-Paternity Problems.*

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2. General formulae on probabilities of proving non-paternity.

We now enter into our main discourse. Let us consider, as usual, an inherited character consisting of m allelomorphic genes A_i $(i=1, \ldots, m)$ with an equilibrium distribution given by (1.1). Though the case of mixed mother-child combination is rather general, we first treat, as a model, that of pure one; the former will be discussed in a subsequent section.

In general, we denote by

$$V(ij;\ hk) \tag{2.1}$$

the probability of proving non-paternity of a putative father, chosen at random with respect to type, against a given pair of a mother A_{ij} and her child A_{hk}. Among such quantities, only those are significant in which h or k coincides with at least one of i and j; otherwise, they may be regarded, according to impossibility of mother-child combinations, as to be equal to unity, but such a convention will become really a matter of indifference in the following lines. Let us again, as in (1.1) of IV, denote by $\pi(ij;\ hk)$ the probability of appearing of such a mother-child combination. The probability of the composed event that such a combination arises and then the proof of non-paternity can be established, is thus given by the product

$$P(ij\ ;\ hk)=\pi(ij;\ hk)V(ij;\ hk). \tag{2.2}$$

It vanishes unless h or k coincides with at least one of i and j, regardless of the determination of value of (2.1), since then $\pi(ij;\ hk)$ so does.

If we sum up the probabilities $P(ij;\ hk)$ over all possible types A_{hk} of children, then we get the *sub-probability* of proving non-paternity against the type A_{ij} of mother, which will be denoted by

$$P(ij)=\sum_{h,\,k}P(ij;\ hk). \tag{2.3}$$

The probability of proving non-paternity against a fixed mother of type A_{ij} is then given by

$$P(ij)\,/\,\bar{A}_{ij}\,. \tag{2.4}$$