17. Note on Dirichlet Series (IX). Remarks on J. J. Gergen-S. Mandelbrojt's Theorems.

By Chuji TANAKA.

Mathematical Institute, Waseda University, Tokyo.

(Comm. by Z. SUETUNA, M.J.A., Feb. 12, 1952.)

(1) **Introduction.** Let us put

$$(1.1)\quad F(s)=\sum_{n=1}^{\infty}a_n\exp(-\lambda_n s)\qquad (s=\sigma+it,\ 0\leqq\lambda_1<\lambda_2\ldots<\lambda_n\to\infty).$$

If (1.1) is simply convergent in the whole plane, then (1.1) defines an integral function. Now we shall begin with

Definition. *Let (1.1) be simply convergent in the whole plane. Suppose that (1.1) assumes every value, except perhaps two (∞ included), infinitely many times, in any angular domain:* $|\arg(s-s_0)-\theta|<\varepsilon$, *where* s_0*: fixed point,* ε*: any positive number. Then* $\arg(s-s_0)=\theta$ *is called Julia's direction with respect to* s_0*. For brevity, we denote it by* $J(s_0:\theta)$*-direction.*

In the last part of their interesting note ([1] theorems V-VII), J. J. Gergen-S. Mandelbrojt established the existence of $J(0:\theta)$-directions under some assumptions. In this note, we shall prove the existence of $J(s_0:\theta)$-directions under hypotheses somewhat different from their ones.

(2) **Theorem I.** In this section, we shall prove

Theorem I. Let (1.1) be simply convergent in the whole plane, and not be a constant. Then, for any given point $s_0=\sigma_0+it_0$*, there exist two* $J(s_0:\pm\pi/2)$*-directions, provided that (1.1) is uniformly convergent for* $\sigma_0-\alpha\leqq\sigma$*, where* α*: sufficiently small positive constant.*

From this theorem immediately follows

Corollary. Let (1.1) be uniformly convergent in the whole plane, and not be a constant. Then, for any given point s_0*, there exist two* $J(s_0:\pm\pi/2)$*-directions.*

Formerly the author proved this corollary under the absolute convergence in the whole plane, but recently Prof. A. Wintner kindly remarked to him that this corollary is valid.

In order to establish theorem 1, we need some lemmas.

Lemma I. (H. Bohr, [2] p. 49) *If (1.1) is uniformly convergent for* $\sigma_0\leqq\sigma$*, then to any bounded domain* Δ *interior to this half-plane, and to any given* $\varepsilon(>0)$*, corresponds a sequence of numbers* $\{\tau_p\}$ *such that, for any* s *contained in* Δ *we have*

$$|F(s+i\tau_p)-F(s)|<\varepsilon\qquad (p=\pm1,\ \pm2,\ldots),$$

where