41. Probability-theoretic Investigations on Inheritance. VIII$_2$. Further Discussionson Non-Paternity Problems.

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(Comm. by T. FURUHATA, M.J.A., March 12, 1952.)

2$^{\text{bis}}$. Sub-probability with respect to a type of wife.

We have hitherto considered a probability with respect to each fixed couple. If a frequency of mating is also to be taken into account, the probability has only to be multiplied by a respective mating-frequency; the resulting probability will be, corresponding to (2.2) of VII, denoted by

$$W(ij,\ hk) \equiv \bar{A}_{ij}\bar{A}_{hk}U(ij,\ hk) \qquad (i, j, h, k=1,\dots, m;\ i \leqq j;\ h \leqq k). \tag{2.11}$$

We put further, corresponding to (2.3) of VII,

$$W(ij) = \sum_{h,k} W(ij,\ hk), \tag{2.12}$$

the summation extending over all possible sets of suffices, i.e., $h, k=1,\dots, m;\ h \leqq k$. The quantity $W(ij)$ thus defined represents the sub-probability of proving non-paternity with respect to the fixed type A_{ij} of wives. As already noticed in § 1, it must coincide just with the quantity introduced in (2.3) of VII; namely, the identical relation holds:

$$W(ij) = P(ij). \tag{2.13}$$

We shall now verify in a direct manner the validity of the identity (2.13), to make sure. For that purpose, we first consider a homozygotic wife A_{ii}. We then get, corresponding to (2.12) of VII,

$$W(ii) = W(ii, ii) + \sum_{h\neq i}(W(ii, ih) + W(ii, hh)) + \sum_{h,k\neq i}{}' W(ii, hk). \tag{2.14}$$

Substituting the respective values of (2.11) obtained by (2.2) to (2.5) into the right-hand side of (2.14) and then remembering the first relation (1.16) of VII, we get

$$\begin{aligned} W(ii) &= p_i^4(1-p_i) + \sum_{h\neq i}(2p_i^3p_h(1-p_i-p_h) + p_i^2p_h^2(1-p_h)) \\ &\qquad + \sum_{h,k\neq i}{}' 2p_i^2p_hp_k(1-p_h-p_k) \\ &= p_i^2\{p_i^2(1-p_i) + 2p_i((1-p_i)^2-(S_2-p_i^2)) + S_2 - p_i^2 - (S_3-p_i^3) \\ &\qquad + 1 - 2S_2 - 2p_i(1-p_i-S_2) - (S_2-2S_3) + 2p_i^2(1-2p_i)\} \\ &= p_i^2(1-2S_2+S_3), \end{aligned} \tag{2.15}$$