54. Probability-theoretic Investigations on Inheritance. IX₄. Non-Paternity Concerning Mother-Children Combinations.

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7^{bis}. Probability against at least one child.

The results on partial sums with respect to (7.2) can also be obtained in a direct manner by means of the following table concerning the set of deniable types of a man and its probability (7.1) against each triple.

Mother	1st child \ 2nd child	A_{ii}	A_{ih}	A_{ik}
A_{ii}	A_{ii}	$A_{ab}(a, b\neq i)$ $(1-p_i)^2$	$A_{ab}(\neq A_{ih})$ $1-2p_ip_h$	$A_{ab}(\neq A_{ik})$ $1-2p_ip_k$
	A_{ih}	$A_{ab}(\neq A_{ih})$ $1-2p_ip_h$	$A_{ab}(a, b\neq h)$ $(1-p_h)^2$	$A_{ab}(\neq A_{hk})$ $1-2p_hp_k$
	A_{ik}	$A_{ab}(\neq A_{ik})$ $1-2p_ip_k$	$A_{ab}(\neq A_{hk})$ $1-2p_hp_k$	$A_{ab}(a, b\neq k)$ $(1-p_k)^2$

Mother	1st child \ 2nd child	A_{ii}	A_{jj}	A_{ij}	A_{ih}, A_{jh}	A_{ik}, A_{jk}
A_{ij}	A_{ii}	$A_{ab}(a, b\neq i)$ $(1-p_i)^2$	$A_{ab}(\neq A_{ij})$ $1-2p_ip_j$	$A_{ab}(a, b\neq i)$ $(1-p_i)^2$	$A_{ab}(\neq A_{ih})$ $1-2p_ip_h$	$A_{ab}(\neq A_{ik})$ $1-2p_ip_k$
	A_{jj}	$A_{ab}(\neq A_{ij})$ $1-2p_ip_j$	$A_{ab}(a, b\neq j)$ $(1-p_j)^2$	$A_{ab}(a, b\neq j)$ $(1-p_j)^2$	$A_{ab}(\neq A_{jh})$ $1-2p_jp_h$	$A_{ab}(\neq A_{jk})$ $1-2p_jp_k$
	A_{ij}	$A_{ab}(a, b\neq i)$ $(1-p_i)^2$	$A_{ab}(a, b\neq j)$ $(1-p_j)^2$	$A_{ab}(a, b\neq i, j)$ $(1-p_i-p_j)^2$	$A_{ab}(\neq A_{ih}, A_{jh})$ $1-2(p_i+p_j)p_h$	$A_{ab}(\neq A_{ik}, A_{jk})$ $1-2(p_i+p_j)p_k$
	A_{ih} or A_{jh}	$A_{ab}(\neq A_{ih})$ $1-2p_ip_h$	$A_{ab}(\neq A_{jh})$ $1-2p_jp_h$	$A_{ab}(\neq A_{ih}, A_{jh})$ $1-2(p_i+p_j)p_h$	$A_{ab}(a, b\neq h)$ $(1-p_h)^2$	$A_{ab}(\neq A_{hk})$ $1-2p_hp_k$
	A_{ik} or A_{jk}	$A_{ab}(\neq A_{ik})$ $1-2p_ip_k$	$A_{ab}(\neq A_{jk})$ $1-2p_jp_k$	$A_{ab}(\neq A_{ik}, A_{jk})$ $1-2(p_i+p_j)p_k$	$A_{ab}(\neq A_{hk})$ $1-2p_hp_k$	$A_{ab}(a, b\neq k)$ $(1-p_k)^2$

We first derive the relations concerning (7.5) which correspond to (2.6) to (2.10) or (4.13) to (4.17). The results are as follows:

(7.10) $\tilde{J}(ii; ii)=\frac{1}{2}p_i^3(2-2(1+S_2)p_i-p_i^2+3p_i^3),$

(7.11) $\tilde{J}(ii; ih)=\frac{1}{2}p_i^2p_h(2-2(1+S_2)p_h-p_h^2+3p_h^3);$

(7.12) $\tilde{J}(ij; ii)=\frac{1}{4}p_i^2p_j(4-4(1+S_2)p_i-2p_i(p_i+p_j)+6p_i^3+p_ip_j(p_i+2p_j)),$