53. Probability-theoretic Investigations on Inheritance. IX$_3$. Non-Paternity Concerning Mother-Children Combinations.

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5. Decomposition of the probability J with regard to types of children.

We now proceed to decompose the whole probability J in (4.25) into sub-probabilities with respect to pairs of children types. Corresponding to (3.1), we denote by

$$K(hk, fg) = \sum_{i \leq j} Q(ij; hk, fg) \tag{5.1}$$

the sub-probability of proving non-paternity against both children (A_{hk}, A_{fg}).

In order to calculate the value of (5.1), it will again be convenient to consider an excess of (3.1). In view of (4.6), an inequality

$$K(hk, fg) \leqq H(hk, fg) \tag{5.2}$$

holds in general, while, in particular, a useful equality

$$K(fg, fg) = H(fg, fg) \tag{5.3}$$

holds good. The results corresponding to (3.2) to (3.10) are as follows:

$$K(ff, ff) = H(ff, ff), \tag{5.4}$$

$$K(hh, ff) = H(hh, ff) - \tfrac{1}{4}p_f^2 p_h^3(2 - 2p_f - p_h) \qquad (h \neq f), \tag{5.5}$$

$$K(hf, ff) = H(hf, ff) - \tfrac{1}{2}p_f^2 p_h^2(1 + p_f)(2 - 2p_f - p_h) \qquad (h \neq f), \tag{5.6}$$

$$K(hk, ff) = H(hk, ff) - \tfrac{1}{4}p_f^2 p_h p_k(2(1 - p_f)(p_h + p_k) - (p_h^2 + p_k^2)) \qquad (h, k \neq f;\ h \neq k); \tag{5.7}$$

$$K(ff, fg) = H(ff, fg) - \tfrac{1}{4}p_f^3 p_g(2 + p_f - p_f^2 - (4 + p_f)p_g + 2p_g^2) \qquad (f \neq g), \tag{5.8}$$

$$K(fg, fg) = H(fg, fg), \tag{5.9}$$

$$\begin{aligned} K(hf, fg) = H(hf, fg) - \tfrac{1}{4}p_f p_g p_h(&p_f^2(2 - p_f - 2p_g) \\ &+ (2 + 8p_f - 2p_g - 5p_f^2 - 10p_f p_g)p_h - (1 + 5p_f)p_h^2) \\ &(f \neq g;\ h \neq f, g), \end{aligned} \tag{5.10}$$

$$K(hh, fg) = H(hh, fg) - \tfrac{1}{2}p_f p_g p_h^3(2 - (p_f + p_g) - p_h) \qquad (f \neq g;\ h \neq f, g), \tag{5.11}$$

$$\begin{aligned} K(hk, fg) = H(hk, fg)& \\ - \tfrac{1}{2}p_f p_g p_h p_k&((2 - p_f - p_g)(p_h + p_k) - (p_h^2 + p_k^2)) \\ &(f \neq g;\ h, k \neq f, g;\ h \neq k). \end{aligned} \tag{5.12}$$