46. On the Theory of Continuous Information.

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1. Introduction. The information theory was first expounded by C.E. Shannon and is now attacked by some authors. I suppose, however, it is sufficiently completed in the case of the discrete system. But it seems to me somewhat vaguely in the continuous system. The most difficulty is that, continuous information lacks the unit of measure. Gabor[1] and some authors have noticed that there was such a relation between the bandwidth and the time duration as the uncertainty relation in quantum mechanics. I think, this relation plays an essential rôle in continuous information.

The definition of the entropy of the system will be most properly defined as the measure of the uncertainty which this system owns, or the power to transmit the information. In other words, information will be defined as the measure of the decrease of uncertainty. When we have some information about the system in question, the uncertainty of the system must be decreased. Assuming the certainty to correspond to the zero uncertainty, the measure of uncertainty must have difinite sign. In any case, we could not have more information than that which the first *a priori* uncertainty has. Therefore, I think, we can not consider the negative value of the entropy when defining the entropy to have the positive sign[2].

2. The entropy produced by the linear transformations. Now we consider the simplest case where the number of the random variables is only one, and the following transformation from x to a new variable y:

$$y=ax, \tag{2.1}$$

where a is a positive constant.

The entropy of the system is defined by Shannon as follows,

$$H(x)=-\int p(x)\log p(x)\,dx. \tag{2.2}$$

The new entropy of the system which is induced by (2.1) is given by

1) Phil. Mag. 41 (1949) p. 1161.

2) Rather we may consider the negative value of the information which increases the uncertainty of the system than the first expected one. For example, if we had the exact information which denied any law in physics, we should give up the every concepts that were introduced by this law and the uncertainty would increase.