45. *On the Stability of the Satellite Systems.*

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The motion of our satellite, the Moon, was successfully studied by Hill on the basis of his ingenious theory on the periodic solution of the problem[1]. Suppose that the Sun at infinity moves round the Earth as the origin in a circular and uniform motion, and that the Moon moves in the rotating co-ordinate plane with the x-axis pointing always towards the Sun. Hill has chosen a periodic orbit for the Moon in this rotating axes with two arbitrary integration constants as his intermediary orbit for the actual motion of the Moon. The actual orbit has been obtained from this intermediary orbit by the superposition of small periodic terms and possibly of small empirical secular terms which are supposed to be due to the tidal friction in the main part. Thus the principal feature on the stability of the motion of the Moon can be judged by the behavior of the intermediary orbit, provided that the amplitudes of the small periodic terms remain always small within finite limits.

Let x and y be the rectangular axes, the x axis always pointing to the Sun which moves round the Earth as the origin with the mean motion n' in the xy-plane. The equations of motion of the Moon are

$$\frac{dx^2}{dt^2}-2n'\frac{dy}{dt}+\left[\frac{\mu}{r^3}-3n'^2\right]x=0,$$

$$\frac{d^2y}{dt^2}+2n'\frac{dx}{dt}+\frac{\mu}{r^3}y=0,$$

where μ is the sum of the masses of the Moon and the Earth multiplied by the constant of gravitation. The integral, the so-called Jacobi integral, can be found easily:

$$\frac{1}{2}\left\{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right\}=\frac{\mu}{r}+\frac{3}{2}n'^2x^2-C,$$

with the constant of integration C. The left-hand member should be positive or zero. Hence the motion should occur within the region, where

$$\frac{\mu}{(x^2+y^2)^{1/2}}+\frac{3}{2}n'^2x^2-C\geqq 0.$$

1) Hill, Amer. J. Math., **5**, 129 and 245, 1873; Collected Math. Works, Vol. I, 284, 1905; Acta Math., **8**, 1, 1886; Collected Math. Works, Vol. I, 243, 1905.