61. Probability-theoretic Investigations on Inheritance. X_3. Non-Paternity Concerning Mother-Child-Child Combinations.

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(Comm. by T. FURUHATA, M.J.A., May 13, 1952.)

5. Illustrative examples, recessive genes being existent.

General discussions developed in the preceding sections have exclusively concerned genotypes. If, however, phenotypes containing recessive genes are taken as basic unit, the circumstances will become somewhat different. As frequently mentioned, if recessive genes are existent, genotype of an individual cannot necessarily be determined from its phenotype in a unique manner. Corresponding to the fact that a directly observable character is a phenotype, probabilities a posteriori of possible types of father against every given mother-child combination are also to be determined based upon phenotypes, what will be illustrated by various human blood types.

We first consider ABO blood type. Now, given a mother-child combination $(O\,;\,O)$, and types except O alone may be possible as type of father, of which the probabilities a priori for O, A, B have to be taken as

$$\bar{O}=r^2, \qquad \bar{A}=p(p+2r), \qquad \bar{B}=q(q+2r),$$

respectively. On the other hand, the mating $O\times O$, $A\times O$, $B\times O$, orders being taken into account, produce a child O with probabilities

$$1, \qquad \frac{r}{p+2r}, \qquad \frac{r}{q+2r},$$

respectively. Hence, due to Bayes' theorem, probabilities a posteriori of types of father being O, A, B are respectively given by

$$Z(O,\,O;\,O)=1\cdot\bar{O}\Big/\Big(1\cdot\bar{O}+\frac{r}{p+2r}\cdot\bar{A}+\frac{r}{q+2r}\cdot\bar{B}\Big)=r,$$

$$Z(A,\,O;\,O)=\frac{r}{p+2r}\cdot\bar{A}\Big/\Big(1\cdot\bar{O}+\frac{r}{p+2r}\cdot\bar{A}+\frac{r}{q+2r}\cdot\bar{B}\Big)=p,$$

$$Z(B,\,O;\,O)=\frac{r}{q+2r}\cdot\bar{B}\Big/\Big(1\cdot\bar{O}+\frac{r}{p+2r}\cdot\bar{A}+\frac{r}{q+2r}\cdot\bar{B}\Big)=q.$$

In similar ways, the remaining probabilities a posteriori will be determined. The results, together with those similarly obtained on