58. On the Induced Characters of a Group.

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This short note is a preliminary report for the theory of induced characters of a group. The detailed proofs will be given elsewhere. The present study is closely related to the papers Brauer [1] and [3].

1. Let $\mathfrak{G}$ be a group of finite order $g=q^a g'$ where q is a prime number and $(g', q)=1$ and let $\mathfrak{Q}$ be a fixed q-Sylow-subgroup of $\mathfrak{G}$. Let $C_1, C_2, \dots, C_n$ be the classes of conjugate elements in $\mathfrak{G}$. Further let $C_1, C_2, \dots, C_h$ be the classes of conjugate elements which contain the elements in $\mathfrak{Q}$. We denote by $Q_1=1, Q_2, \dots, Q_h$ $(Q_i \in \mathfrak{Q})$ a complete system of representatives for the classes $C_i (i=1, 2, \dots, h)$. Let $g_i=g/n_i$ be the number of elements in C_i, so that n_i is the order of the normalizer $\mathfrak{N}(Q_i)$ of Q_i in $\mathfrak{G}$. We set $n_i=q_i n_i'$ where $(n_i', q)=1$. q_i is called the q-part of n_i. Let $\varsigma_1, \varsigma_2, \dots, \varsigma_n$ and $\vartheta_1, \vartheta_2, \dots, \vartheta_m$ be distinct irreducible characters of $\mathfrak{G}$ and $\mathfrak{Q}$. In what follows we shall always take ς_1 and ϑ_1 to be the characters of the 1-representations of $\mathfrak{G}$ and $\mathfrak{Q}$. If ϑ_ν^* is the character of $\mathfrak{G}$ induced from ϑ_ν, then we have the following Frobenius formulas

$$(1) \qquad \begin{cases} \varsigma_\mu(Q) = \sum_\nu r_{\mu\nu}\vartheta_\nu(Q) & \text{(for } Q \text{ in } \mathfrak{Q}) \\ \vartheta_\nu^*(G) = \sum_\mu r_{\mu\nu}\varsigma_\mu(G) & \text{(for } G \text{ in } \mathfrak{G}), \end{cases}$$

where

$$(2) \qquad r_{11}=1, \quad r_{1\nu}=0 \quad (\nu\neq 1).$$

As is well known, the rank of $M=(r_{\mu\nu})$ is h. We can prove, by the similar way as in Brauer [3][1], the following

Lemma 1. *$M=(r_{\mu\nu})$ contains a minor of degree h which is not divisible by q.*

We set

$$R_1 = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1h} \\ r_{21} & r_{22} & \dots & r_{2h} \\ \dots & \dots & \dots & \dots \\ r_{h1} & r_{h2} & \dots & r_{hh} \end{pmatrix}.$$

Then we may assume without restriction that

1) We can somewhat simplify Brauer's original proof.