# 84. Probability-theoretic Investigations on Inheritance. XII$_2$. Probability of Paternity.

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## 3. Paternity on two-children family.

Similar problems as above will also be discussed with respect to two-children family. Let a fixed mother-children combination $(A_{ij}; A_{hk}, A_{fg})$ be given, and $C_1$ be a cause that a presented man is really a father of the children and $C_2$ be another cause that he is not their father. We suppose here again that the probabilities a priori of these mutually exclusive causes are both equal to 1/2. The probability of an event that, under the cause $C_1$, a mating $A_{ab}\times A_{ij}$ produces the children $A_{hk}$ and $A_{fg}$ has already outlined in § 3 of IV, which will be denoted by

$$(3.1)\qquad \lambda(ab,\ ij;\ hk,\ fg).$$

On the other hand, under the cause $C_2$, a mother $A_{ij}$, together with a common father, produces children $A_{hk}$ and $A_{fg}$ with the probability

$$(3.2)\qquad \pi(ij; hk, fg)/\bar{A}_{ij}.$$

Hence, in view of the Bayes' theorem, for a given mother-children combination $(A_{ij}; A_{hk}, A_{fg})$, the probability a posteriori of a man $A_{ab}$ to be a true father, i.e., his *probability of paternity*, is expressed by

$$(3.3)\qquad \varLambda(ij; hk, fg; ab)=\frac{\lambda(ab,\ ij; hk, fg)}{\lambda(ab,\ ij; hk, fg)+\pi(ij; hk, fg)/\bar{A}_{ij}}.$$

The value of the last expression is determined for every possible quadruple as follows; different letters indicating different genes.

$$(3.4)\qquad \begin{aligned}
&\varLambda(ii;\ ii,\ ii;\ ii)=\frac{2}{2+p_i(1+p_i)}, && \varLambda(ii;\ ii,\ ii;\ ih)=\frac{1}{1+2p_i(1+p_i)};\\
&\varLambda(ii;\ ii,\ ih; ih)=\frac{1}{1+2p_ip_h}; && \varLambda(ii; ih,\ ih; hh)=\frac{2}{2+p_h(1+p_h)},\\
&\varLambda(ii; ih,\ ih; ih)=\frac{1}{1+2p_h(1+p_h)}, && \varLambda(ii; ih,\ ih; hk)=\frac{1}{1+2p_h(1+p_h)};\\
&\varLambda(ii; ih,\ ik; hk)=\frac{1}{1+2p_hp_k};
\end{aligned}$$