74. On a Theorem of K. Yosida.

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1. A topic in the theory of partial differential equations which has received attention of recent years is the question of the behaviour at infinity of solutions which satisfy a null condition on the interior boundary of an infinite region, but which do not vanish identically. For the ordinary wave-equation we have the radiation condition of Sommerfeld, with its electromagnetic analogue. Analogous results have also been established for parabolic equations. Recently K. Yosida (Proc. Japan Acad., 27, 214–215 (1951)) has considered the equation

$$\Delta h(x)=m(x)\,h(x) \tag{1}$$

in a region R which is a connected domain with smooth boundaries ∂R in an n-dimensional Euclidean space R_n, where $n\geqq 2$. Furthermore in R $m(x)$ is to be continuous and have a positive lower bound m, and ∂R is to lie entirely in the bounded part of R_n. Yosida's theorem then states that if $h(x)$ satisfies the internal boundary condition

$$\partial h/\partial n=0 \text{ on } \partial R, \tag{2}$$

and the order relation at infinity

$$h(x)=O(\exp\,(\alpha r)), \text{ where } \alpha\sqrt{2}<\sqrt{m}, \tag{3}$$

then $h(x)$ must vanish identically. Here I use r (in place of Yosida's $|x|$) to denote the distance from the origin of coordinates.

The aim of this note is to show the condition (3) may, by a slight modification of Yosida's argument, be replaced by what seems to be a best possible result in this direction. Consider namely the special case in which $n=3$, $m(x)=k^2$ where k is a positive constant, and in which ∂R is a sphere, centre the origin. We then have the spherically symmetric solutions

$$k(x)=r^{-1}\exp\,(\pm kr),$$

of which a non-trivial linear combination may be formed so as to satisfy (2). The mildest condition of the type of (3) which will exclude such solutions is

$$h(x)=o(r^{-1}\exp\,(kr)),$$