89. A Generalization of a Theorem of Suetuna on Dirichlet Series.

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Introduction.

Professor Z. Suetuna proved in Tôhoku Math. Journal 27, 1926, 248–257, the following interesting theorem: Let χ_1, χ_2, χ_3 be any three primitive Dirichlet characters, i.e. mappings of the multiplicative group of the rational numbers (mod m), for some integer m, into the unit circle in the complex plane. Let

$$L(s, \chi_i)=\sum_{n=1}^{\infty}\frac{\chi_i(n)}{n^s}\,, \qquad \Re(s)>1$$

be the corresponding Dirichlet L-series.

Theorem 1: *If*

$$Z_3(s)=\prod_{i=1}^{3} L(s, \chi_i)\,, \qquad \Re(s)>1$$

when developed into a Dirichlet series has non-negative coefficients, then

$$(1) \qquad Z_3(s)=\zeta(s)^3$$

or

$$(2) \qquad Z_3(s)=\zeta(s)\zeta_{F_1}(s)$$

or

$$(3) \qquad Z_3(s)=\zeta_{F_2}s)\,,$$

where $\zeta(s)$ *is the Riemann zeta-function,* $\zeta_{F_1}(s)$ *is the Dedekind zeta-function of some quadratic extension of the rational numbers, and* $\zeta_{F_2}(s)$ *is the Dedekind zeta-function of some cubic Abelian extension of the rationals.*

What we propose to prove in the following paper, is that if χ_0, $\chi_1, \ldots, \chi_n$ are any $n+1$ characters (mod m), not necessarily distinct, with at most one of the characters being principal, and if

$$\prod_{j=0}^{n} L(s, \chi_j)$$

has non-negative coefficients, then

$$(4) \qquad \prod_{j=0}^{n} L(s, \chi_j)=\zeta_K(s)$$

where K is a finite Abelian extension of the rationals, and $\zeta_K(s)$ is the corresponding Dedekind zeta-function.