120. Probability-theoretic Investigations on Inheritance. XVI_2. Further Discussions on Interchange of Infants.

By Yûsaku KOMATU.

Department of Mathematics, Tokyo Institute of Technology and
Department of Legal Medicine, Tokyo Medical and Dental University.

(Comm. by T. FURUHATA, M.J.A., Nov. 12, 1952.)

1. Supplementary remarks.

Similarly as remarked in § 6 of XV, the whole probability (2.15) may also be regarded as the probability of an event that, two mothers and their two children being presented, the decision is possible.

An inequality correspondiny to (6.1) *of* XV is valid here also:

$$F_0 \geqq \Psi, \quad \text{i.e.} \quad F_0 \geqq \tfrac{1}{2} F \geqq \Psi; \tag{3.1}$$

the reason being quite the same as there.

We shall now compare the probabilities derived in the preceding section with the corresponding ones, previously obtained in § 5 of XV. If the detection of interchange is possible with reference to both pairs of mother-child combinations, it is of course possible with reference to both triples of mating-child combinations. Hence, we conclude *an inequality*

$$G_0(ij) \geqq F_0(ij) \qquad (i \leqq j). \tag{3.2}$$

This inequality can also be verified directly by means of explicit expressions of its both sides. Namely, making use of (5.27) and (5.28) of XV and (2.3) and (2.4), we see that

$$\begin{aligned} G_0(ii) - F_0(ii) &= p_i^3(2(1-2S_2+S_3) - p_i + 2p_i^2 - p_i^3) \\ &= p_i^3\Big(2\sum_{h \neq i} p_h(1-p_h)^2 + p_i(1-p_i)^2\Big) \geqq 0, \end{aligned} \tag{3.3}$$

$$\begin{aligned} G_0(ij) - F_0(ij) &= 2p_ip_j(2(1-2S_2+S_3)(p_i+p_j) - (p_i^2+p_j^2) - 2p_ip_j \\ &\quad + 2(p_i^3+p_j^3) - (p_i^4+p_j^4) + 4p_i^2p_j^2) \\ &= 2p_ip_j\Big(2(p_i+p_j)\sum_{h \neq i,\, j} p_h(1-p_h)^2 \\ &\quad + (p_i+p_j)^2(1-p_i-p_j)^2 + 2p_ip_j(p_i+p_j)(1-p_i-p_j) \\ &\quad + 2p_i^2p_j^2\Big) \geqq 0 \qquad (i \neq j). \end{aligned} \tag{3.4}$$

That an inequality of the same nature

$$G(ij) \geqq F(ij) \qquad (i \leqq j) \tag{3.5}$$

holds good is also a matter of course; this can also be verified in a direct manner. Hence, we see further

$$G \geqq F. \tag{3.6}$$

The general results reduce for $m=2$ to ones concerning MN