116. Probability-theoretic Investigations on Inheritance. XV_2. Detection of Interchange of Infants.

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3. Another method of attack.

Wiener has arrived at his result on the whole probability of detecting interchange of infants, given in (2.16), by calculating the 36 partial probabilities one after another and then summing up them. But, this method will become rapidly troublesome as the number of possible types of an inherited character increases. In fact, if there exist, in general, m^* different phenotypes, then the possible matings amount to $m^*(m^*+1)/2$ kinds so that the combinations of matings to be considered amount to $m^{*2}(m^*+1)^2/4$ in number. For instance, this number is equal to 100 or 441 for $m^*=4$ (ABO blood type) or $m^*=6$ (A_1A_2BO blood type), respectively. Even when the combinations with identically vanishing probability are omitted and further only the half of the remaining combinations are considered in view of symmetry, they amount yet to

$$(3.1) \quad \tfrac{1}{2}\left(\tfrac{1}{4}m^{*2}(m^*+1)^2-\tfrac{1}{2}m^*(m^*+1)\right)=\tfrac{1}{8}(m^*-1)m^*(m^*+1)(m^*+2),$$

which is equal to 45 or 210 for $m^*=4$ or $m^*=6$, respectively.

On account of the reason just stated, we shall now again deal with the problem by *another method* of attack which will directly apply also to general mode of an inherited character.

First, we observe the first mating of $M\times M$. Then, the detection of interchange is possible when and only when the true child of the second mating is not M. Since the child different from M appears with frequency $1-s^2$, we get the partial probability

$$(3.2) \qquad s^4(1-s^2).$$

Similarly, in case of the first mating $N\times N$, the partial probability is given by

$$(3.3) \qquad t^4(1-t^2),$$

and in case of the first mating $M\times N$, it is equal to

$$(3.4) \qquad 2s^2t^2(1-2st).$$

The above three are the cases where the first mating can produce only one type of child.